Reading Course 'Quantum Optics'

PHYS 771

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Assignment 1

1.1. The quantization of the e/m field has been introduced via the vector potential and the conjugate momentum. The electric and magnetic fields can be derived from these quantities (see page 21, 3rd ed., page 31 2nd ed.). Apply this to the plane-wave mode expansion (2.87 and 2.88, either ed.).

1.2. Consider the quantum mechanical state of an optical mode with frequency ω given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |10\rangle\right)$$

which is a superposition of Fock states with photon number 0 and 10.

a) Calculate the mean energy as a function of time.

b) Consider a black box that cannot create or destroy photons, though it can re-direct them. The box works such that whenever a Fock state is entered in the main input with one or more photon, and no photon enters in the second auxiliary input, then the auxiliary output contains one photon, and the main output contains the remaining number of photons. The whole set-up is unitary.
   - What are the conditional states of the conditional states resulting from the input state $|\Psi\rangle$?
   - What is their expectation value of the energy?
   - What is the probability for each event?

c) Repeat the same calculation for the coherent state

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

d) Is $a|\Psi\rangle$, with a being the annihilation operator of the mode, the correct description of the process that takes one photon out of the mode? (Use that $a|\alpha\rangle = \alpha|\alpha\rangle$.)