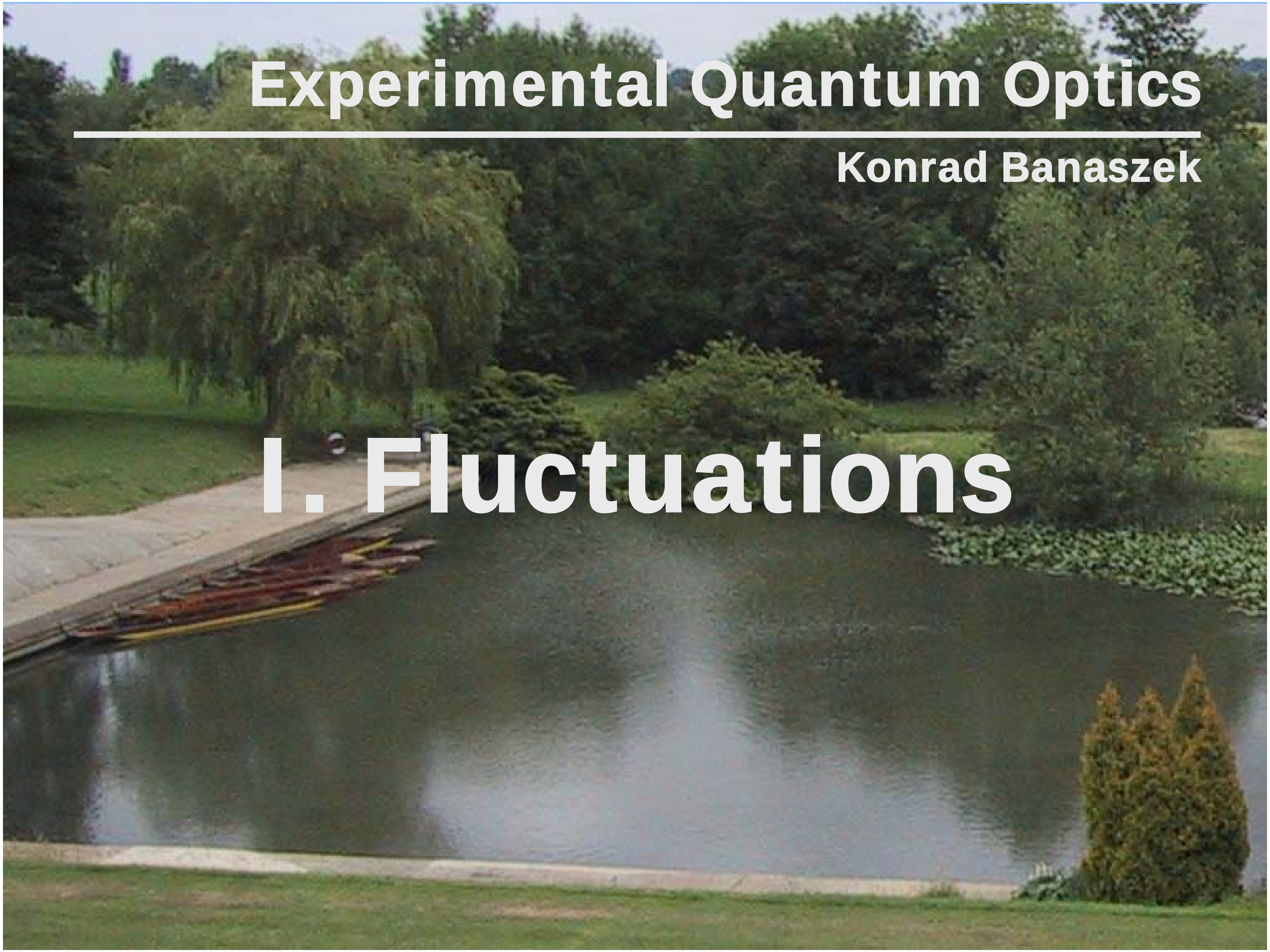
Experimental Quantum Optics

Konrad Banaszek

I. Fluctuations



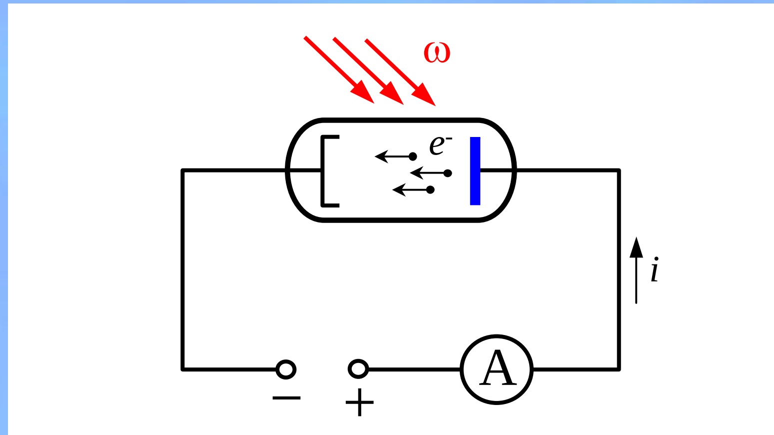
Photoelectric effect

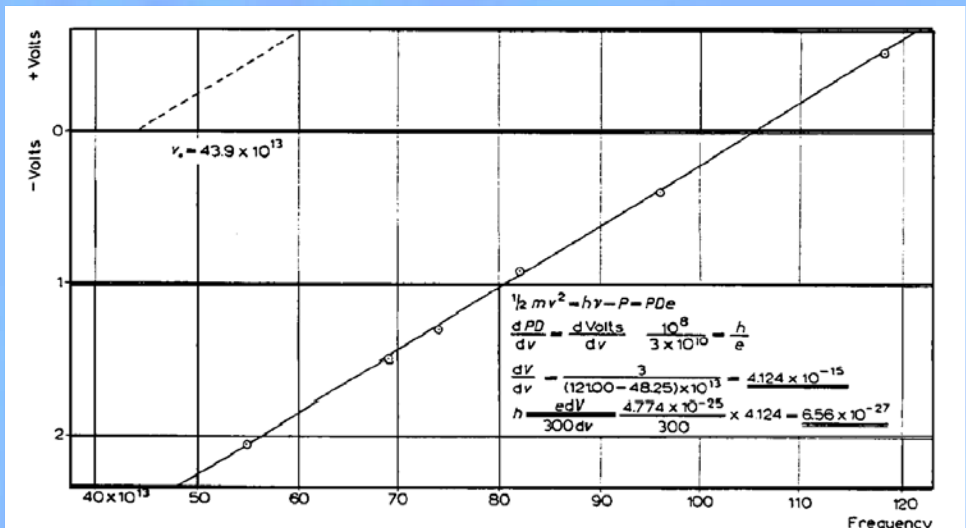
Maximum kinetic energy:

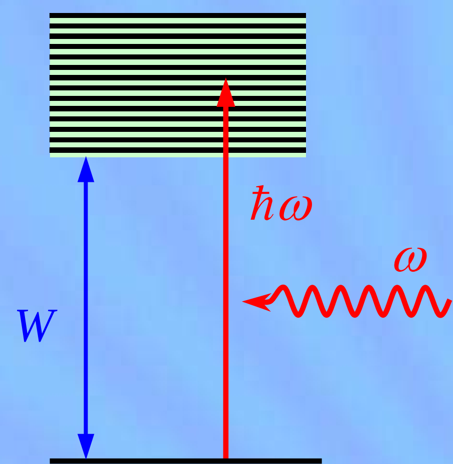
$$E_{\max} = \hbar\omega - W$$

ω – light frequency

W – work function







R. A. Millikan, Phys. Rev. **7**, 355 (1916)

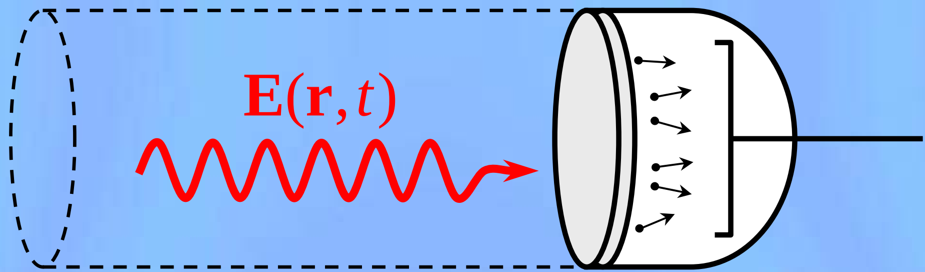
Photocount statistics

Quasi-monochromatic (central frequency ω_0) single-mode field:

$$\mathbf{E}(\mathbf{r},t)=\alpha\mathbf{f}\left(\mathbf{r},t\right)+\text{c.c.}$$

where α is a complex dimensionless field amplitude.

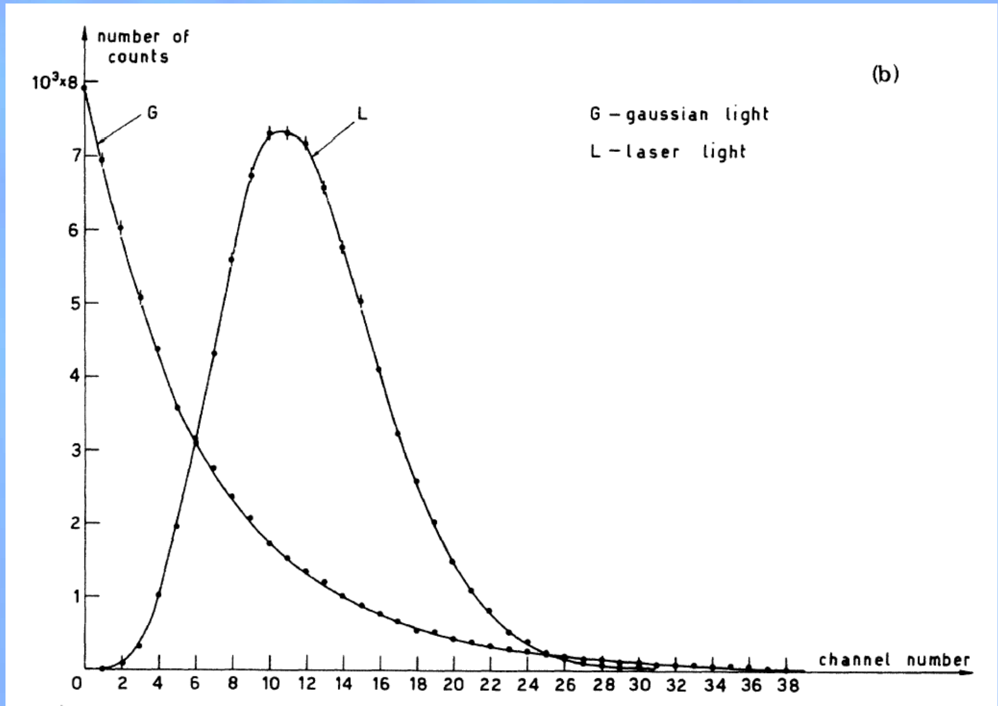
The *mode function* $\mathbf{f}\left(\mathbf{r},t\right)$ is normalized such that the energy contained in the field is $\hbar\omega_0\left|\alpha\right|^2$



Probability of generating n photoelectrons (in the absence of losses) is given by the Poissonian distribution:

$$p_n\left(\alpha\right)=\exp\left(-\left|\alpha\right|^2\right)\frac{\left|\alpha\right|^{2n}}{n!}$$

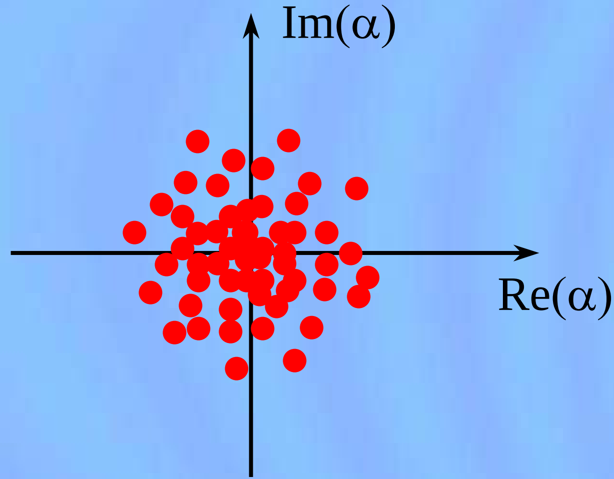
Coherent and thermal light



F. T. Arecchi, Phys. Rev. Lett. **15**, 912 (1965)

Thermal fluctuations:

$$P(\alpha) = \frac{1}{\pi \bar{I}} \exp\left(-|\alpha|^2 / \bar{I}\right)$$



$$p_n = \langle p_n(\alpha) \rangle$$

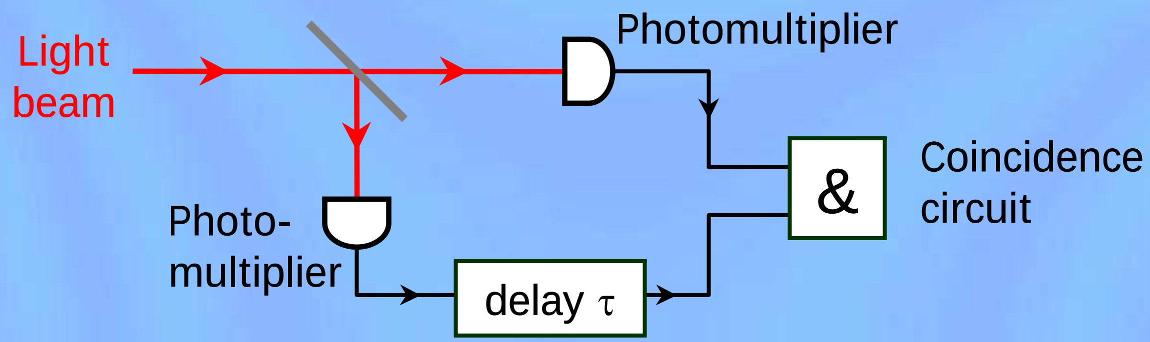
$$= \frac{1}{1+\bar{I}} \left(\frac{\bar{I}}{1+\bar{I}} \right)^n$$

G – Bose-Einstein statistics (thermal light)

L – Poissonian statistics (coherent light)

Intensity correlations

Measurement of the two-time intensity correlation function:



Probability of a click over a short counting time from t to $t+\Delta t$: $p_1 \propto \mathcal{I}(t)\Delta t$

Schwarz inequality:

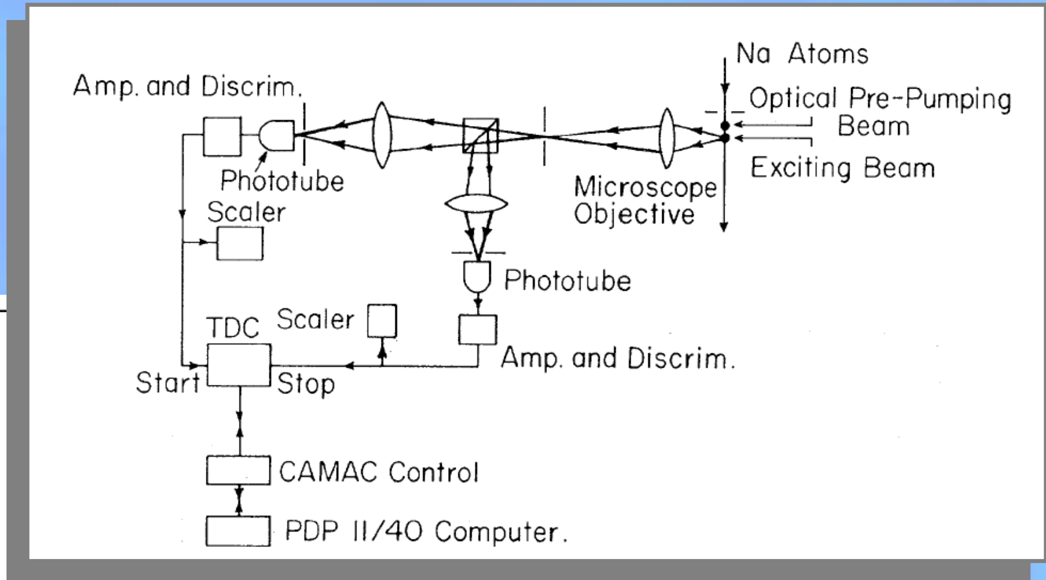
$$\langle \mathcal{I}(t)\mathcal{I}(t+\tau)\rangle \le \sqrt{\langle \mathcal{I}^2(t)\rangle\langle \mathcal{I}^2(t+\tau)\rangle}$$

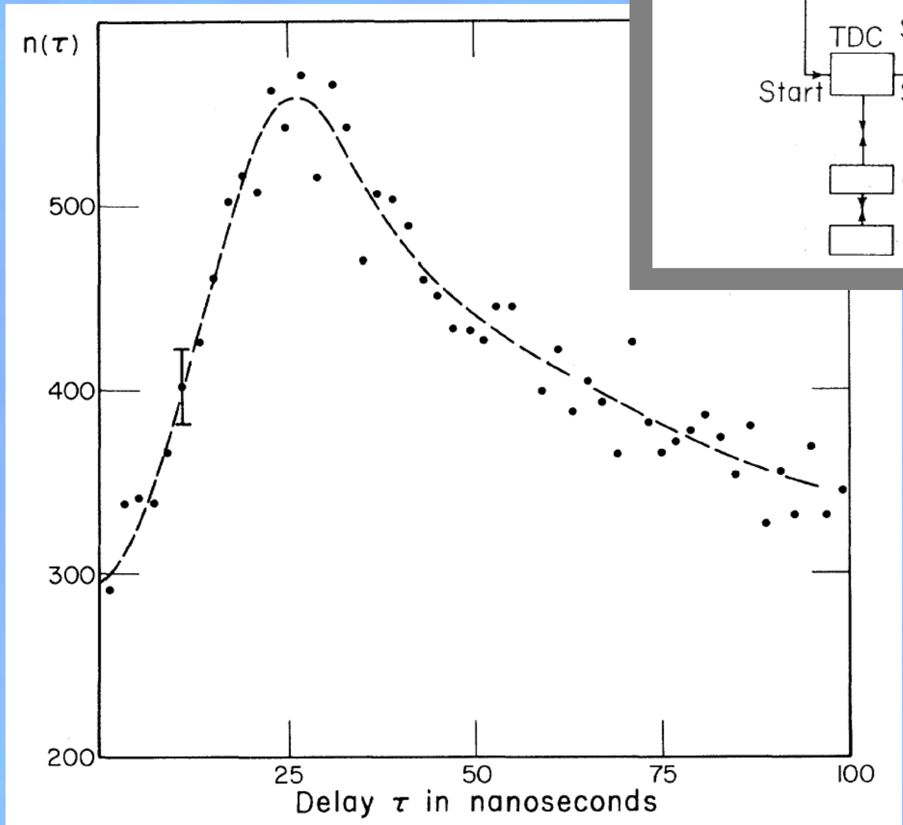
For a stationary source $\langle \mathcal{I}^2(t)\rangle = \langle \mathcal{I}^2(t+\tau)\rangle$ and

$$\langle \mathcal{I}(0)\mathcal{I}(\tau)\rangle \le \langle \mathcal{I}^2(0)\rangle$$

Photon antibunching

H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977)





A single atom can emit only **one photon** at once!

Field quantization

Electric field operator:

$$\hat{\mathbf{E}}(\mathbf{r},t) = \hat{a}\mathbf{f}\left(\mathbf{r},t\right) + \text{H.c.}$$

$$\left(+\text{ other modes}\right)$$

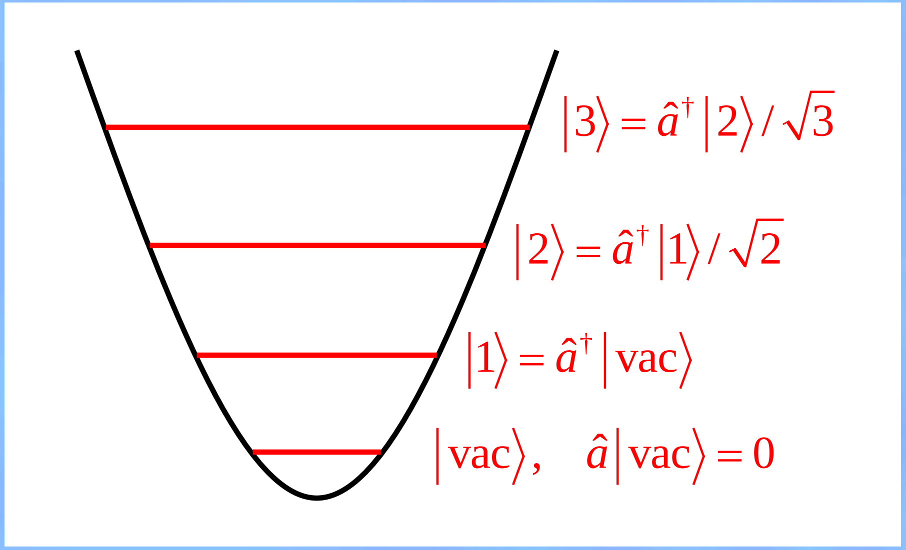
The complex amplitude α becomes an annihilation operator satisfying bosonic commutation relations:

$$[\hat{a}, \hat{a}^\dagger] = 1$$

Eigenstates of $\hat{a}^\dagger \hat{a}$:

$$\hat{a}^\dagger \left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$

$$\hat{a}\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle$$



Quantum states of a light mode

Fock (number) states:

$$|n\rangle = \frac{\left(\hat{a}^\dagger\right)^n}{\sqrt{n!}}|\text{vac}\rangle$$

Coherent states:

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Thermal states:

$$\hat{\rho}_{\text{th}} = \frac{1}{\pi\overline{I}}\int d^2\alpha\, e^{-|\alpha|^2/\overline{I}}\,|\alpha\rangle\langle\alpha| = \frac{1}{1+\overline{I}}\sum_{n=0}^{\infty}\frac{1}{\left(1+1/\overline{I}\right)^n}|n\rangle\langle n|$$

General classical state of radiation:

$$\hat{\rho} = \int d^2\alpha\, P(\alpha)|\alpha\rangle\langle\alpha|$$

Quantum theory of photodetection

Probability of generating n photoelectrons by a perfect photodetector (η=1):

$$p_n = \mathrm{Tr}\left(\hat{\rho}\,|n\rangle\langle n|\right)$$

For a coherent state $|\alpha\rangle$ statistical predictions are the same as of the semiclassical theory:

$$p_n = \left|\langle n|\alpha\rangle\right|^2 = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}$$

Ideal photodetection realises measurement of the photon number operator:

$$\sum_{n=0}^{\infty} n\,|n\rangle\langle n| = \hat{a}^\dagger\hat{a}$$

Moments of the photocount statistics:

$$\sum_{n=0}^{\infty} n^m p_n = \left\langle \sum_{n=0}^{\infty} n^m |n\rangle\langle n| \right\rangle = \left\langle \left(\hat{a}^\dagger\hat{a}\right)^m \right\rangle$$

Photoelectric current fluctuations

Moments of the photoelectron statistics for classical light ($\mathcal{J}=|\alpha|^2$):

$$\langle n\rangle = \sum_{n=0}^{\infty} n p_n = \langle \mathcal{J}\rangle$$

$$\langle n^2\rangle = \sum_{n=0}^{\infty} n^2 p_n = \langle \mathcal{J}^2\rangle + \langle \mathcal{J}\rangle$$

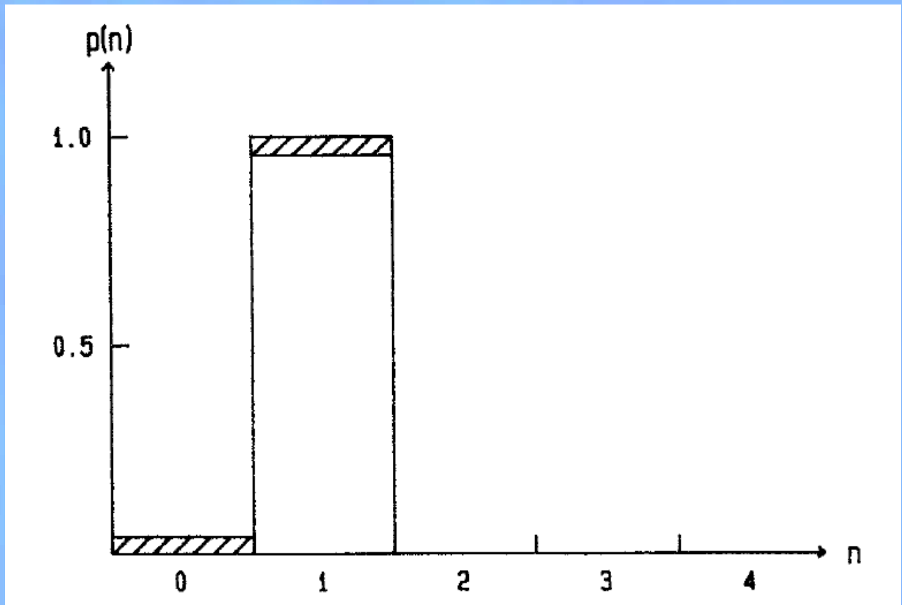
Fluctuations of the photoelectron number:

$$\left\langle (\Delta n)^2 \right\rangle = \langle n^2\rangle - \langle n\rangle^2 = \langle \mathcal{J}^2\rangle + \langle \mathcal{J}\rangle - \langle \mathcal{J}\rangle^2 = \left\langle (\Delta \mathcal{J})^2 \right\rangle + \langle \mathcal{J}\rangle$$

$\left\langle (\Delta \mathcal{J})^2 \right\rangle$: Fluctuations of field intensity

$\langle \mathcal{J}\rangle$: Shot noise

Sub-Poissonian statistics

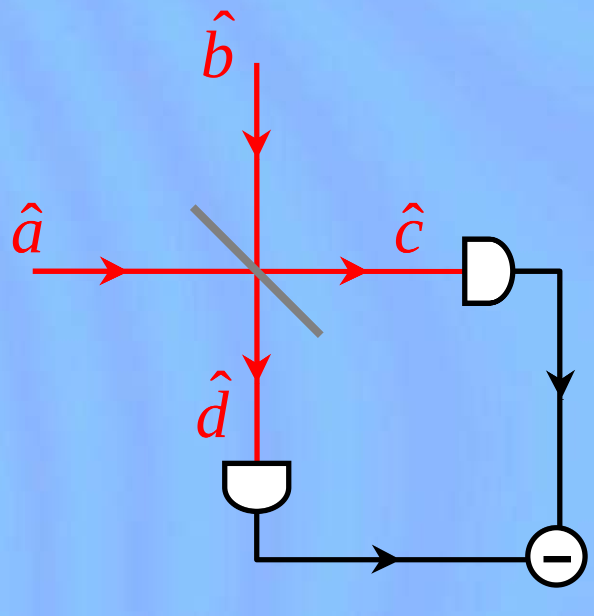


C. K. Hong and L. Mandel,
Phys. Rev. Lett. **56**, 58 (1986).

Observed data	Counting probabilities	Background probabilities	Derived photon probabilities
$N = 6\,000\,000$			
$N(0) = 5\,985\,901$	$P(0) = N(0)/N = 0.997\,65$	$p_b(0) \approx 1$	$p(0) \approx -0.17$ to 0.04
$N(1) = 14\,098$	$P(1) = N(1)/N = 0.002\,35$	$p_b(1) \approx 8.8 \times 10^{-5}$	$p(1) \approx 1.06 \pm 10\%$
$N(2) = 1$	$P(2) = N(2)/N < 10^{-6}$	$p_b(2) \approx 0$	$p(2) \approx 0$

Quantum mechanics helps to suppress fluctuations!

Homodyne detection



50:50 Beam splitter transformation:

$$\hat{c} = \frac{\hat{a}+\hat{b}}{\sqrt{2}} \qquad \hat{d} = \frac{\hat{a}-\hat{b}}{\sqrt{2}}$$

Photoelectron number difference:

$$\hat{I}_- = \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} = \hat{a}\hat{b}^\dagger + \hat{a}^\dagger \hat{b}$$

For the mode $\hat{b}$ in a strong coherent state $|\beta\rangle$ with $\beta = e^{i\theta}|\beta|$ we define:

$$\hat{x}_\theta = \frac{\hat{I}_-}{\sqrt{2}|\beta|} \approx \frac{\hat{a}\beta^* + \hat{a}^\dagger \beta}{\sqrt{2}|\beta|} = \frac{\hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}}{\sqrt{2}}$$

Quadrature fluctuations

For a coherent state $|\alpha\rangle$:

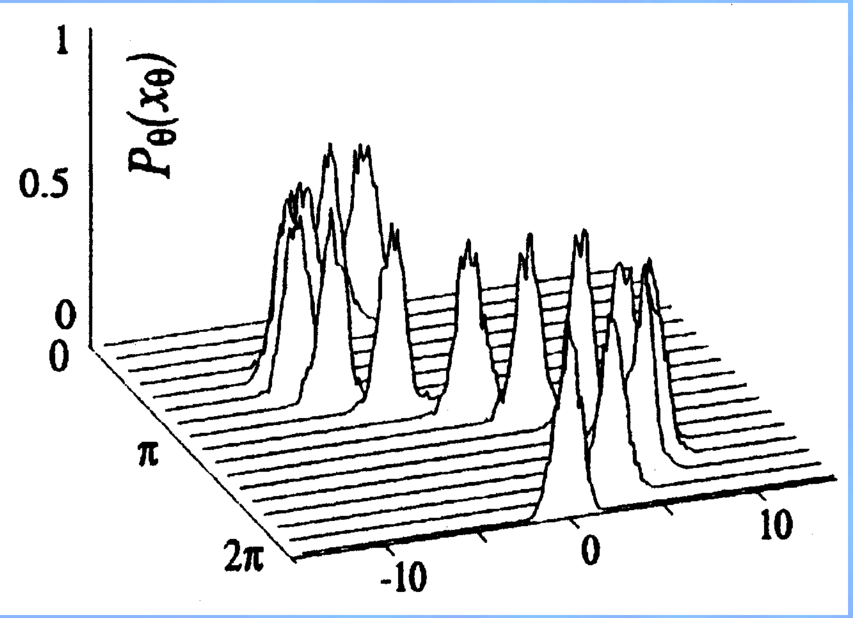
$$\langle\hat{x}_\theta\rangle = \langle\alpha|\hat{x}_\theta|\alpha\rangle = \sqrt{2}\,\mathrm{Re}\left(e^{-i\theta}\alpha\right) = \sqrt{2}\left(\mathrm{Re}\,\alpha\cos\theta + \mathrm{Im}\,\alpha\sin\theta\right)$$

Fluctuations:

$$\left\langle\left(\Delta\hat{x}_\theta\right)^2\right\rangle = \langle\alpha|\hat{x}_\theta^2|\alpha\rangle - \langle\hat{x}_\theta\rangle^2 = \frac{1}{2}$$

For classical states, i.e. coherent states and their statistical mixtures:

$$\left\langle\left(\Delta\hat{x}_\theta\right)^2\right\rangle \geq \frac{1}{2}$$



G. Breitenbach, S. Schiller, and J. Mlynek, Nature **387**, 471 (1997).

Conjugate quadratures

$$\hat{q} = \hat{x}_0 = \frac{\hat{a} + \hat{a}^\dagger}{\sqrt{2}}$$

$$\hat{p} = \hat{x}_{\pi/2} = \frac{\hat{a} - \hat{a}^\dagger}{\sqrt{2}i}$$

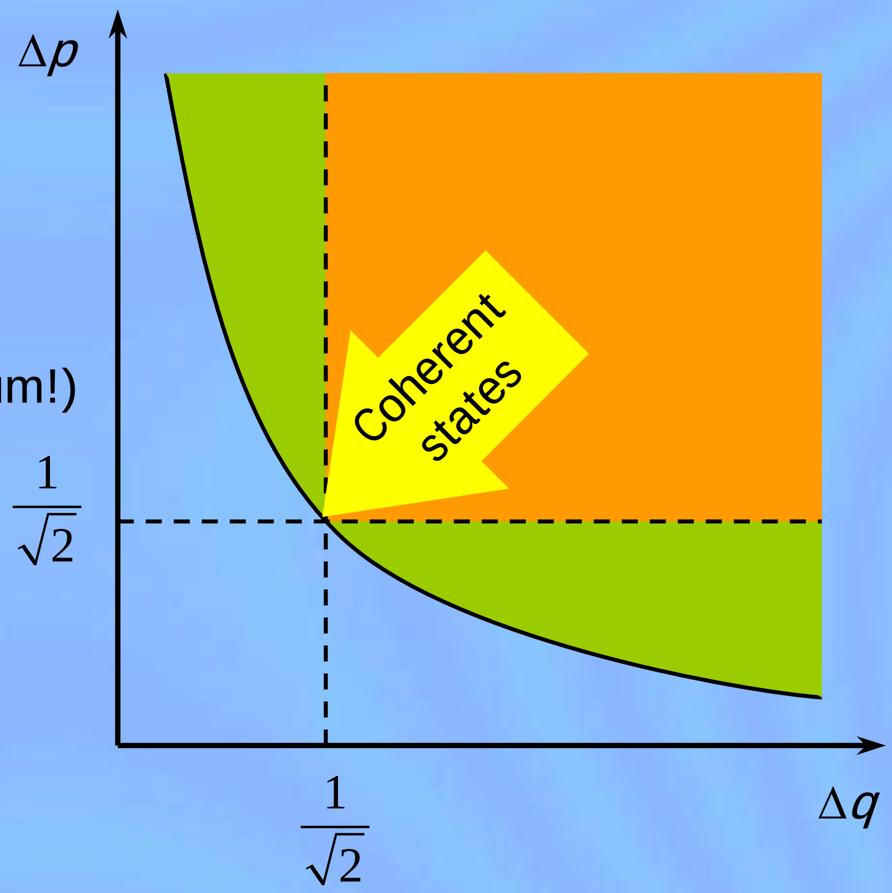
(**NOT** physical position and momentum!)

Commutation relation:

$$\left[\hat{q}, \hat{p}\right] = i$$

Heiseberg uncertainty principle:

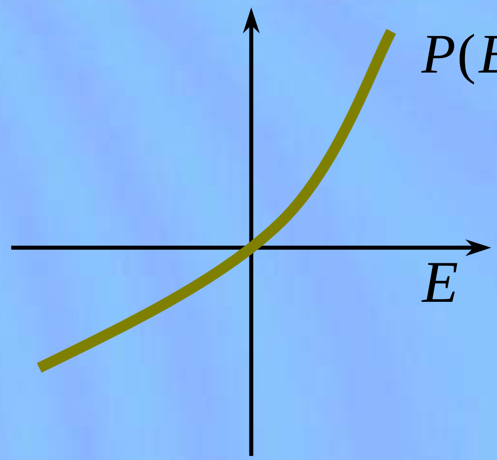
$$\Delta q \cdot \Delta p \geq \frac{1}{2}$$ ( )



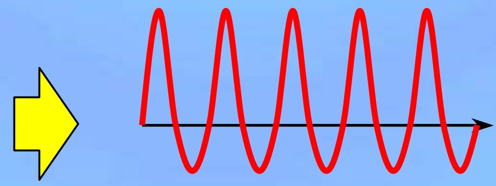
This condition is weaker that the constraints for classical fields:

$$\Delta q \geq \frac{1}{\sqrt{2}} \quad \text{and} \quad \Delta p \geq \frac{1}{\sqrt{2}}$$ ( )

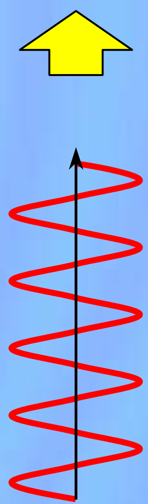
Nonlinear susceptibility

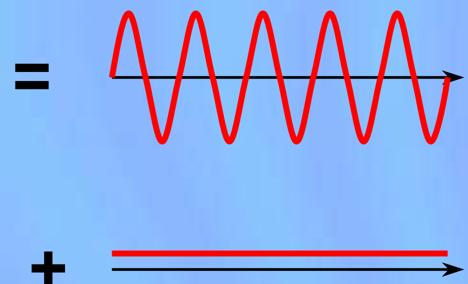


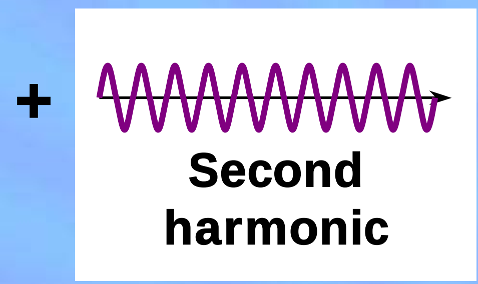
$$P(E) = \chi^{(1)}E + \chi^{(2)}E^2 + \ldots$$



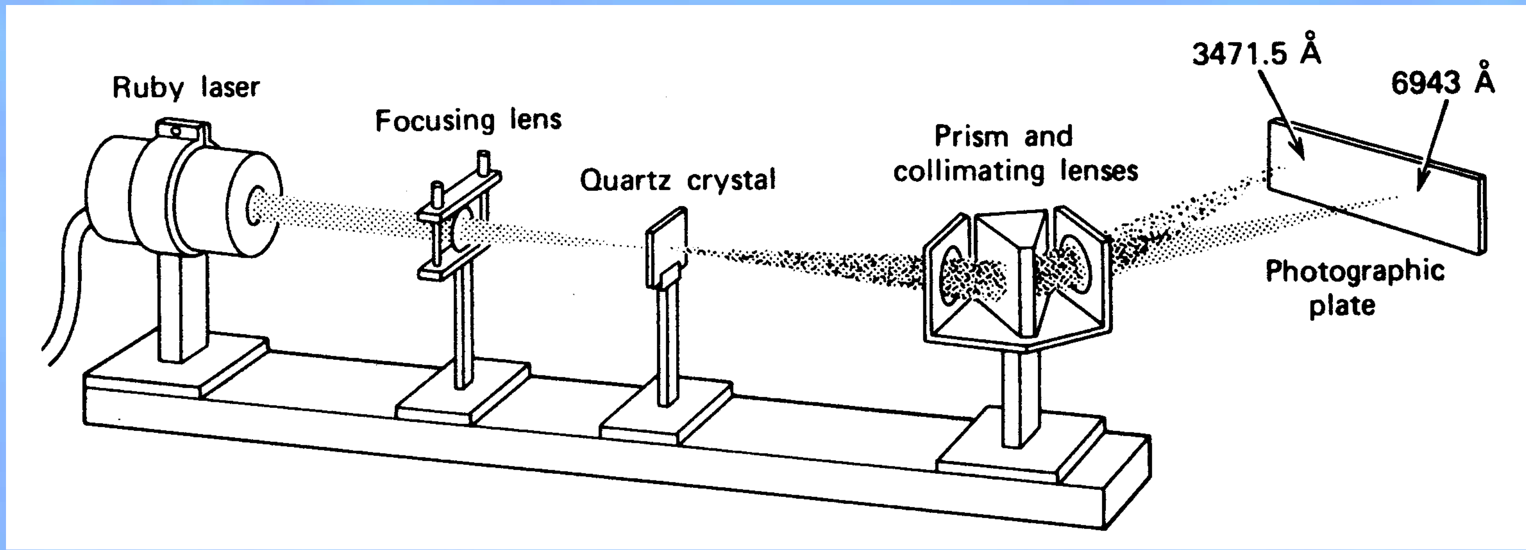
Fourier decomposition:







Second harmonic generation



VOLUME 7, NUMBER 4 PHYSICAL REVIEW LETTERS AUGUST 15, 1961

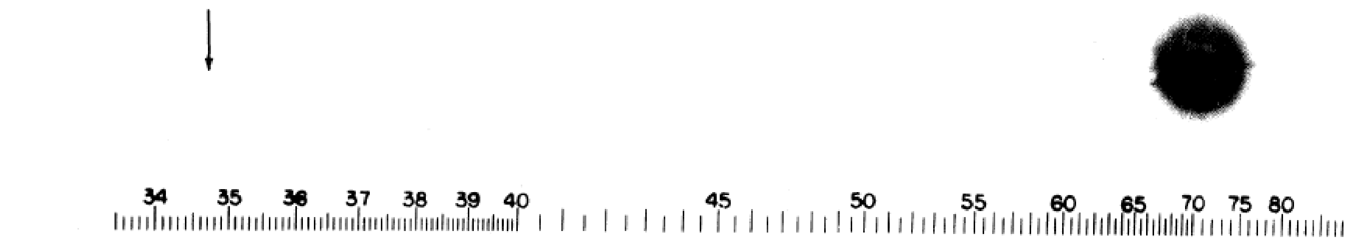


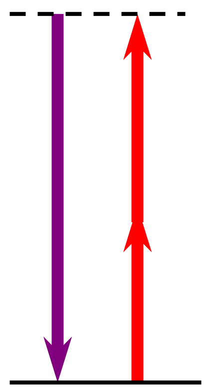
FIG. 1. A direct reproduction of the first plate in which there was an indication of second harmonic. The wavelength scale is in units of 100 A. The arrow at 3472 A indicates the small but dense image produced by the second harmonic. The image of the primary beam at 6943 A is very large due to halation.

P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich, Phys. Rev. Lett. **7**, 118 (1961)

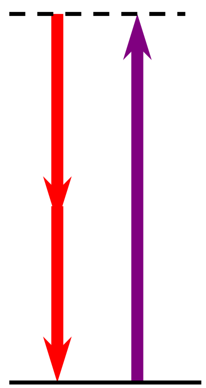
Quantum picture

Interaction Hamiltonian:

$$\hat{H} = \kappa \left(\hat{c}^{\dagger} \hat{a}^{2} + \left(\hat{a}^{\dagger} \right)^{2} \hat{c} \right)$$



Second harmonic generation



Parametric down-conversion

Squeeze operator

Strong classical pump $\hat{c} \to i\gamma$:

$$\hat{H} \approx i\kappa\gamma\left(\left(\hat{a}^\dagger\right)^2 - \hat{a}^2\right)$$

Unitary evolution operator:

$$\hat{U}(r) = \exp\left(-i\hat{H}t/\hbar\right)$$

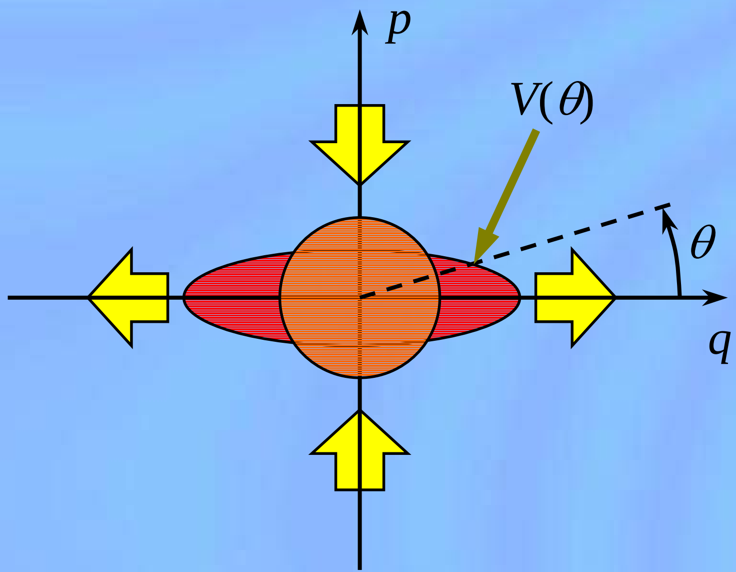
$$= \exp\left[\frac{r}{2}\left(\left(\hat{a}^\dagger\right)^2 - \hat{a}^2\right)\right]$$

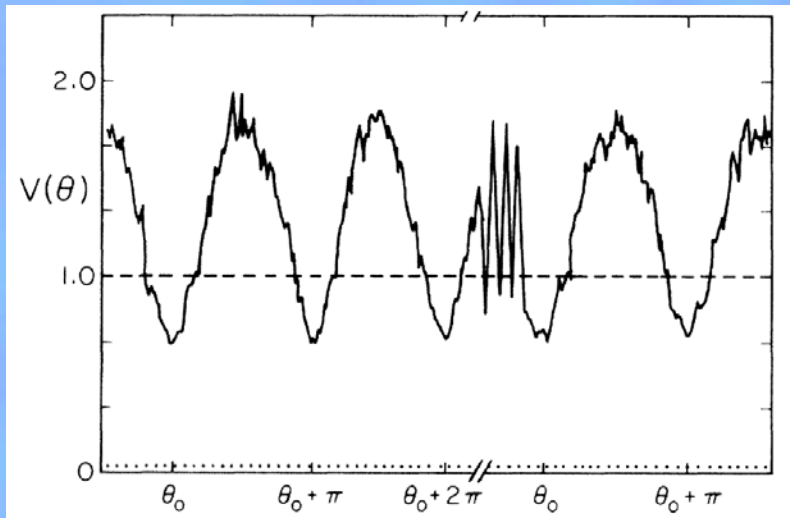
where r is the effective interaction time: $r = 2\gamma\kappa t/\hbar$

Evolution of quadratures:

$$\hat{q}(r) = \hat{U}^\dagger(r)\,\hat{q}\,\hat{U}(r) = e^r\hat{q}$$

$$\hat{p}(r) = \hat{U}^\dagger(r)\,\hat{p}\,\hat{U}(r) = e^{-r}\hat{p}$$

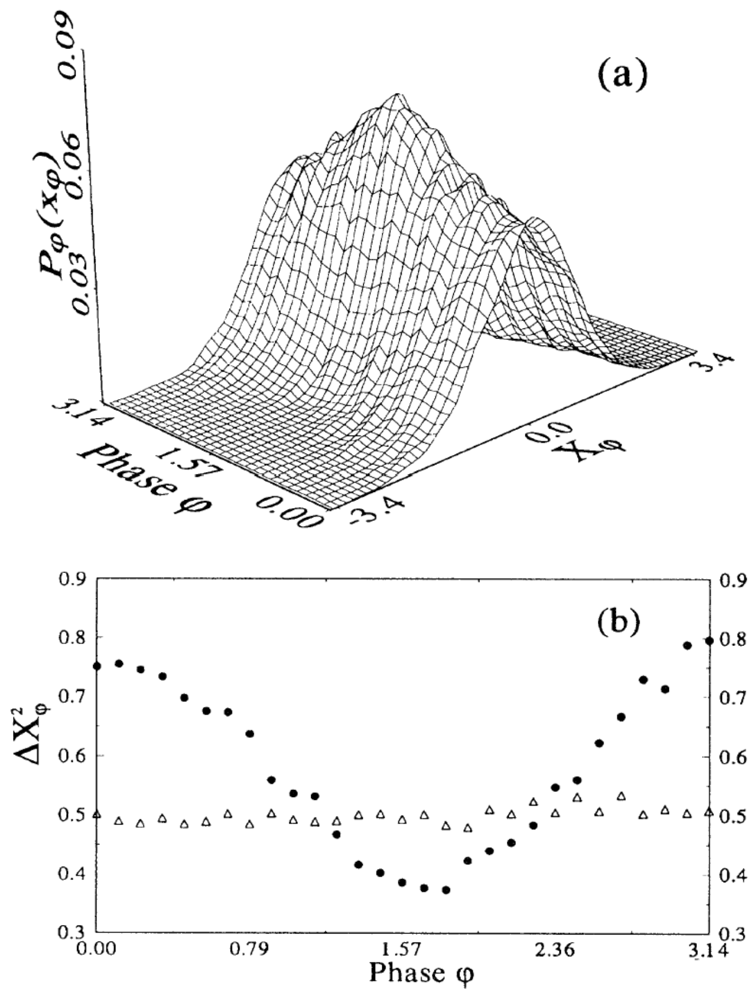


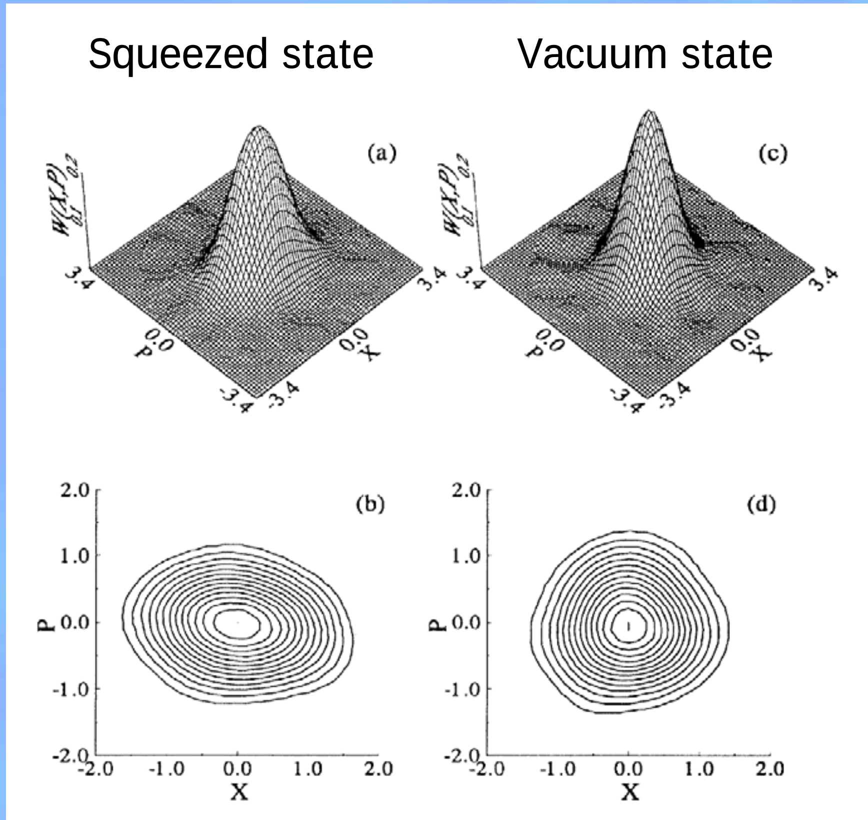


L.-A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986)

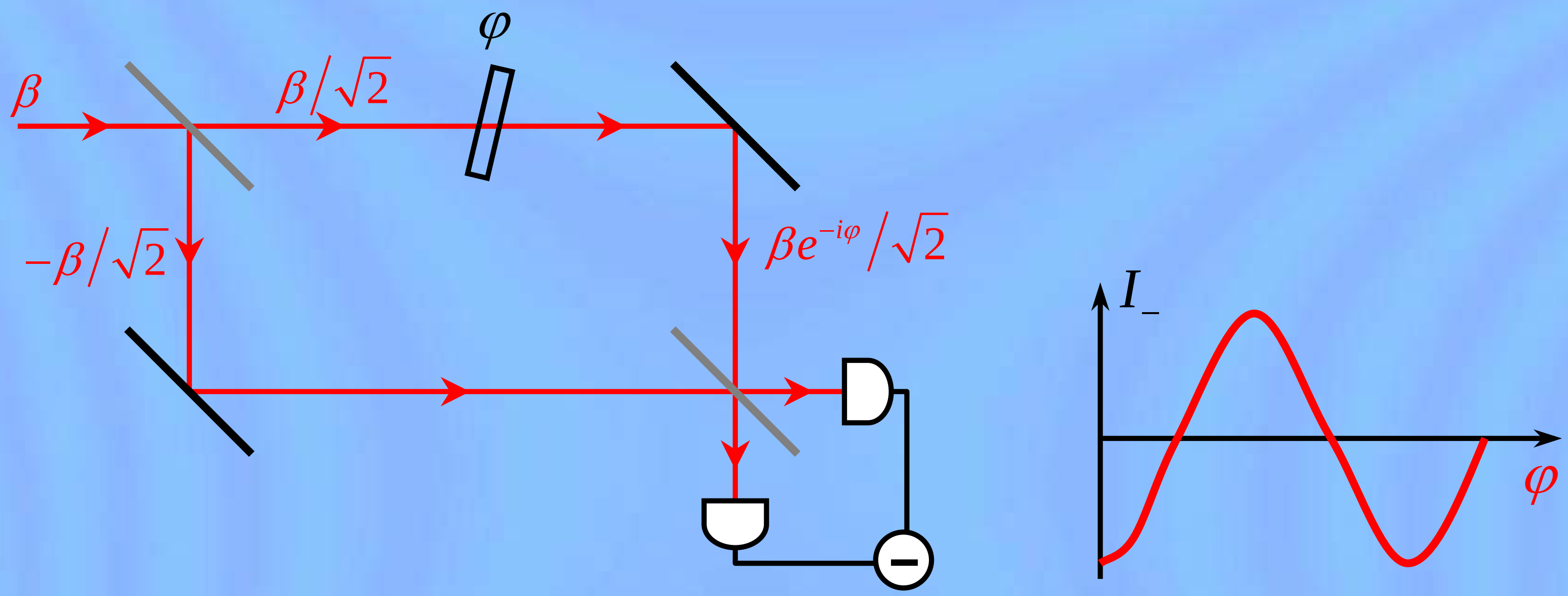
Wigner function

D. T. Smithey, M. Beck,
M. G. Raymer, and A. Faridani,
Phys. Rev. Lett. **70**, 1244 (1993)





Detection of phase shift

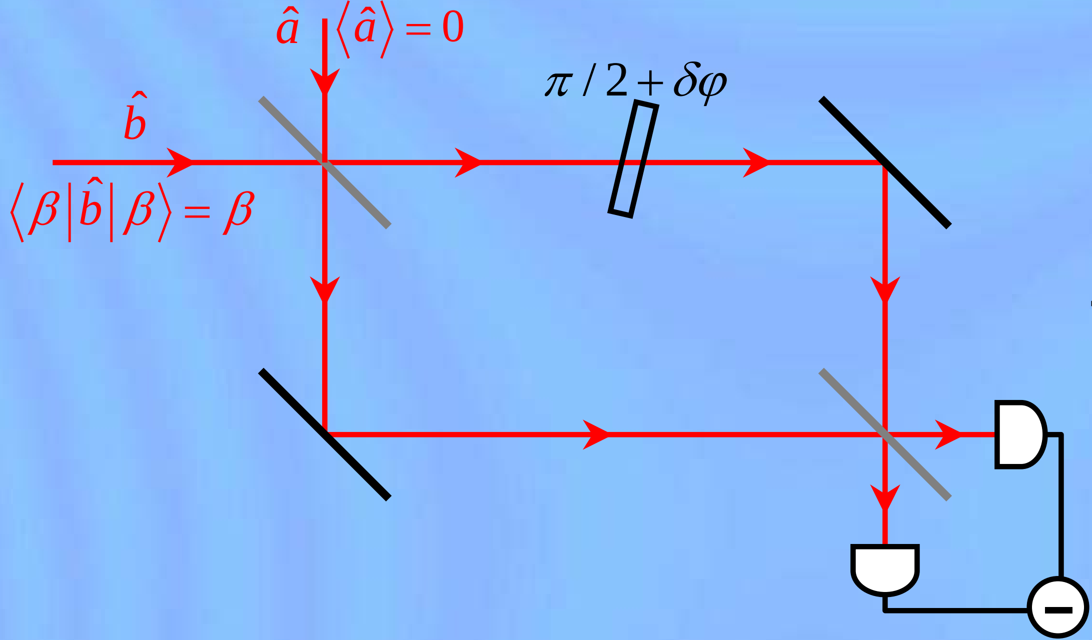


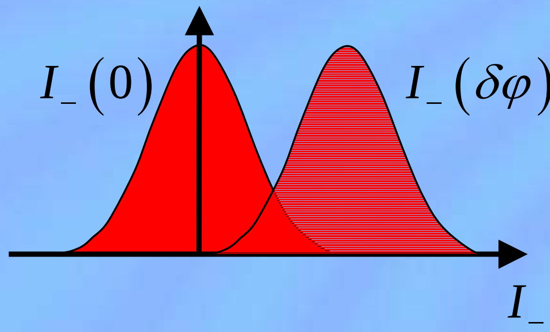
$$I_- = -|\beta|^2 \cos\varphi$$

Best sensitivity for the phase φ around $\pi/2$: when $\varphi=\pi/2+\delta\varphi$

$$I_-(\delta\varphi) = |\beta|^2\,\delta\varphi$$

Shot-noise limit





$$\hat{I}_{-}(\delta\varphi) = \left(\hat{b}^{\dagger}\hat{b} - \hat{a}^{\dagger}\hat{a}\right)\delta\varphi + i\left(\hat{a}\hat{b}^{\dagger} - \hat{a}^{\dagger}\hat{b}\right)$$

Expectation value:

$$\left\langle \hat{I}_{-} \right\rangle \approx |\beta|^{2}\, \delta\varphi$$

Dispersion:

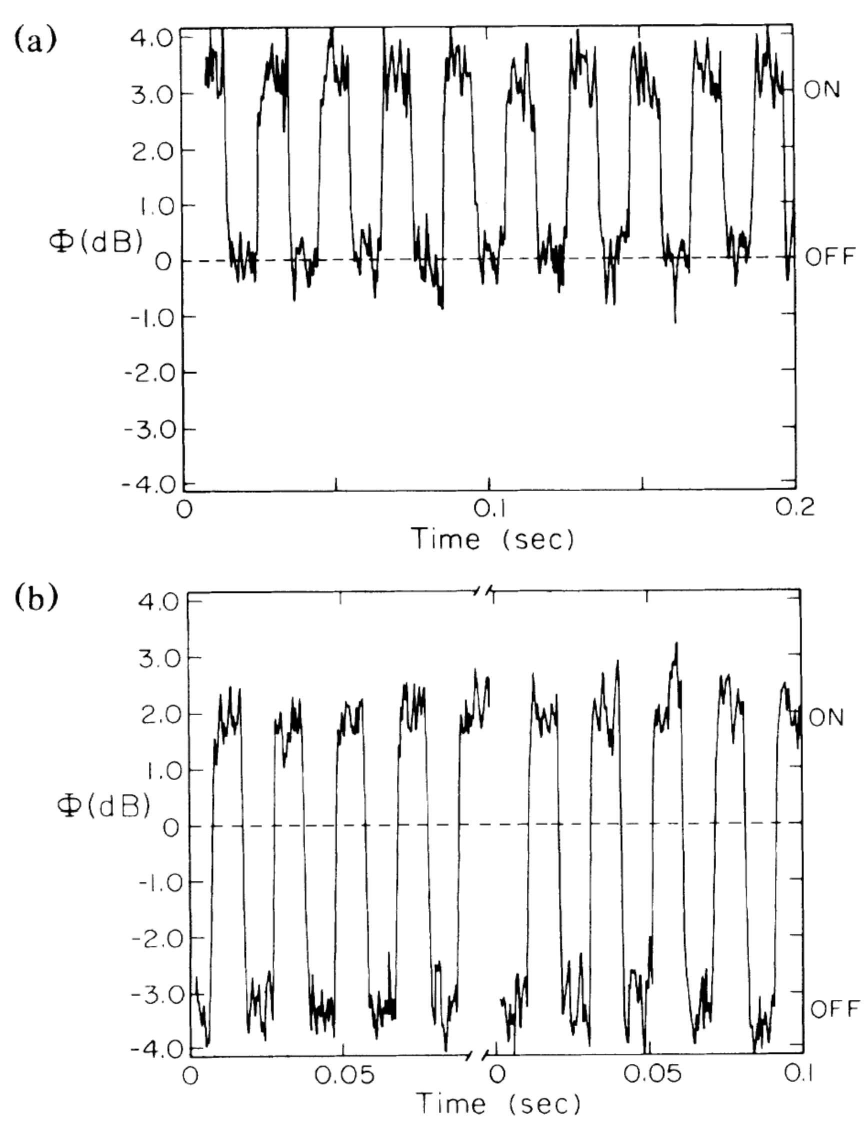
$$\left\langle \left(\Delta \hat{I}_{-}\right)^{2} \right\rangle \approx 2|\beta|^{2} \left\langle \left(\Delta \hat{p}_{a}\right)^{2} \right\rangle$$

Signal-to-Noise Ratio:

$$SNR = \frac{\left\langle \hat{I}_{-} \right\rangle}{\sqrt{\left\langle \left(\Delta \hat{I}_{-}\right)^{2} \right\rangle}} = \frac{|\beta|\,\delta\varphi}{\sqrt{2\left\langle \left(\Delta \hat{p}_{a}\right)^{2} \right\rangle}}$$

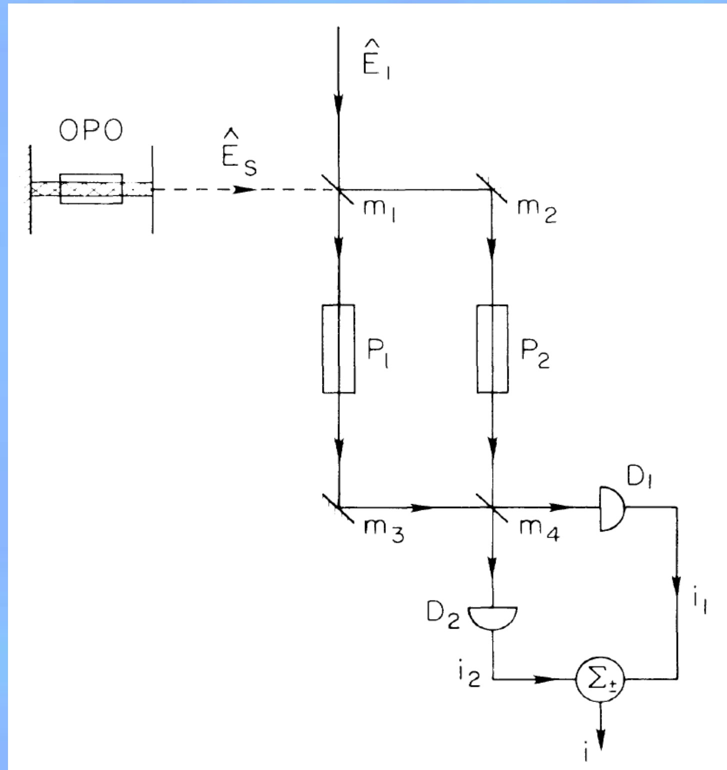
Application of squeezed input is more efficient in terms of the average number of used photons!

Sub-shot-noise interferometry



(a) vacuum input

(b) squeezed input

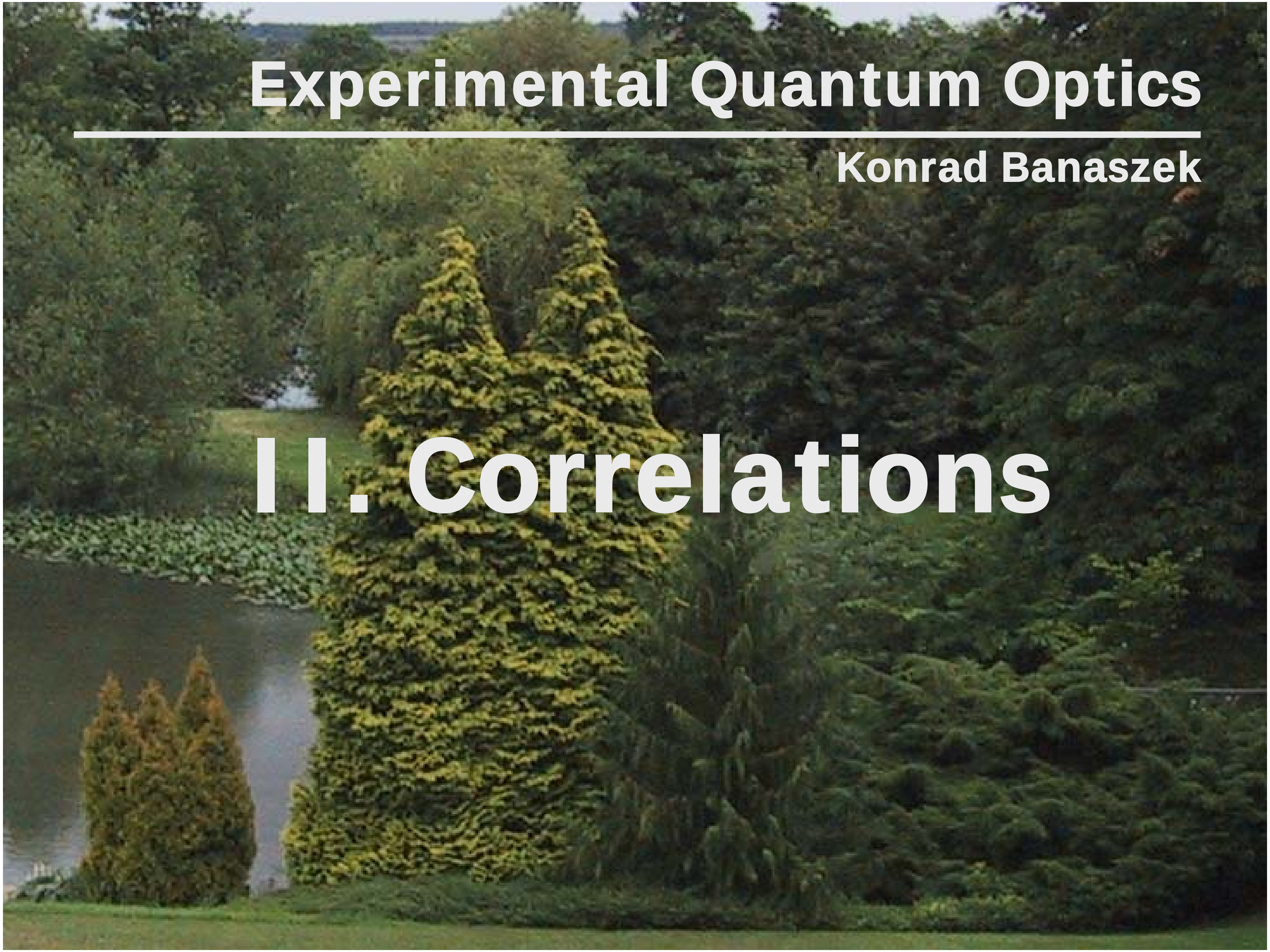


M. Xiao, L.-A. Wu, and H. J. Kimble,
Phys. Rev. Lett. **59**, 278 (1987)

Experimental Quantum Optics

Konrad Banaszek

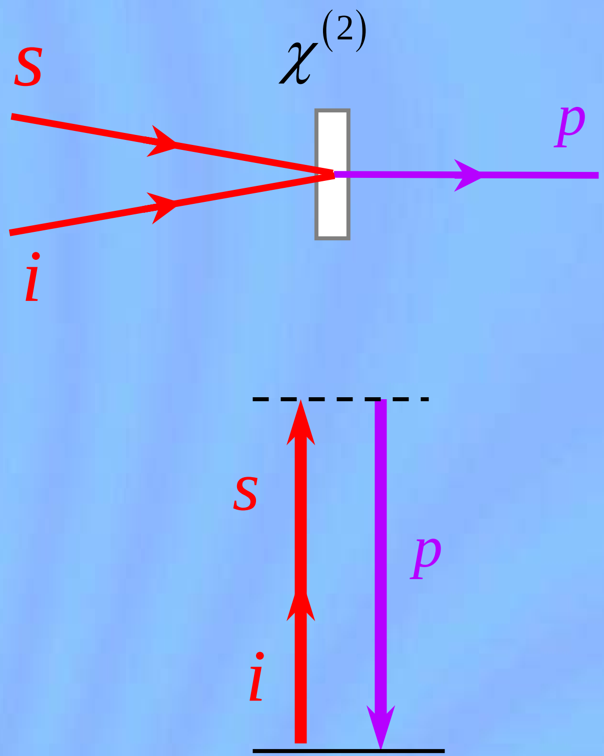
II. Correlations



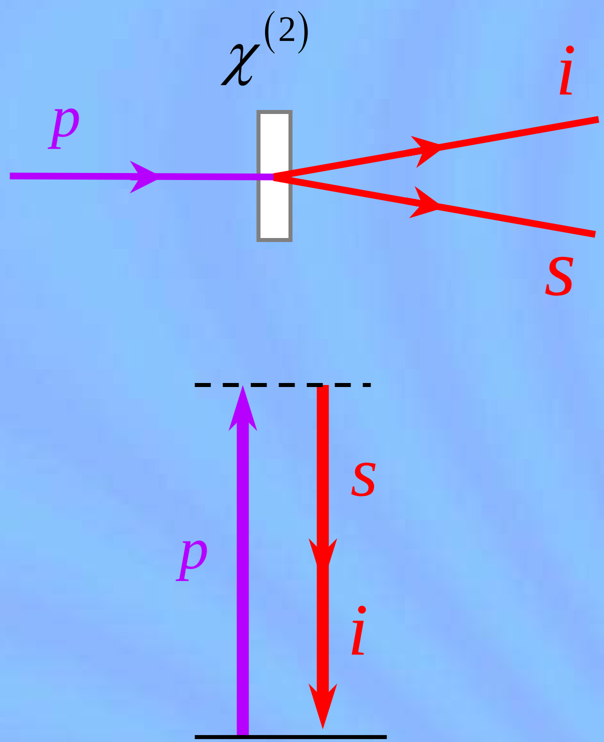
Three-wave mixing

Non-degenerate frequency conversion in $\chi^{(2)}$ crystals:

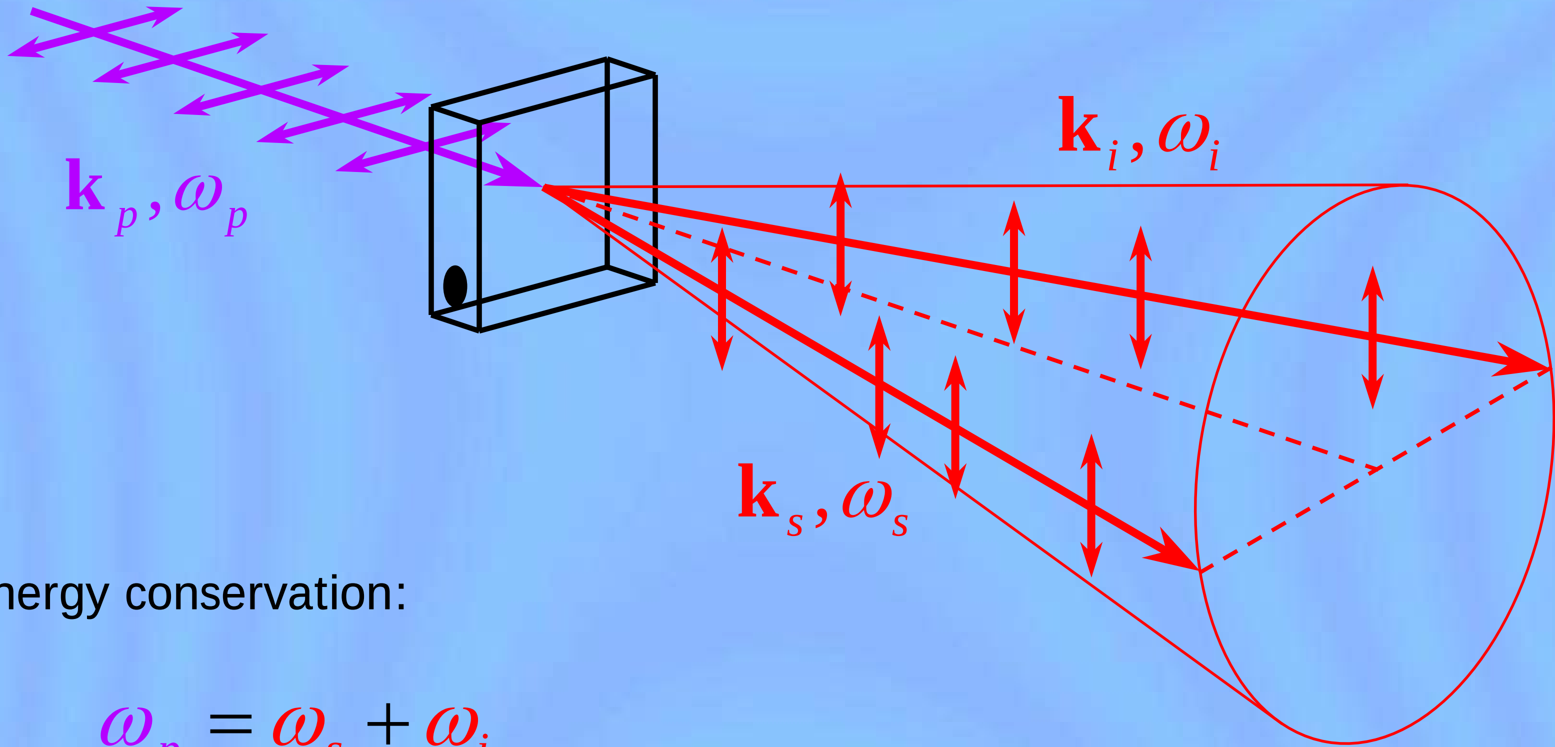
Sum frequency generation:



Parametric down-conversion:



Type-I process



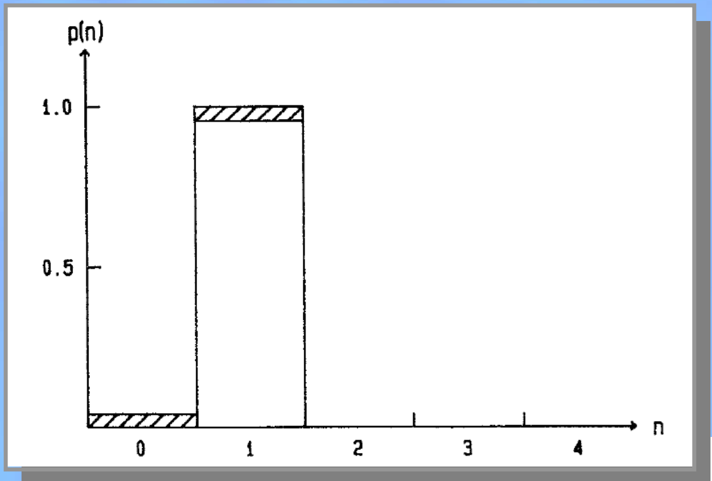
Energy conservation:

$$\omega_p = \omega_s + \omega_i$$

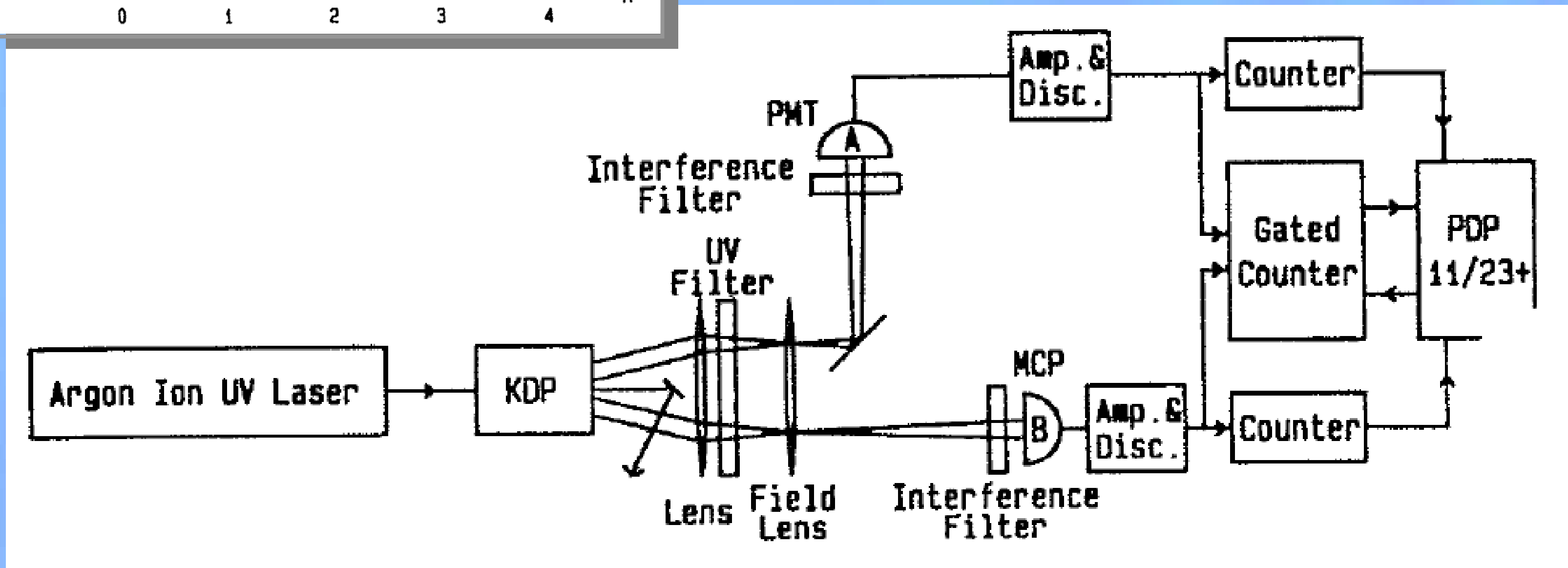
Momentum conservation:

$$\mathbf{k}_p \approx \mathbf{k}_s + \mathbf{k}_i$$

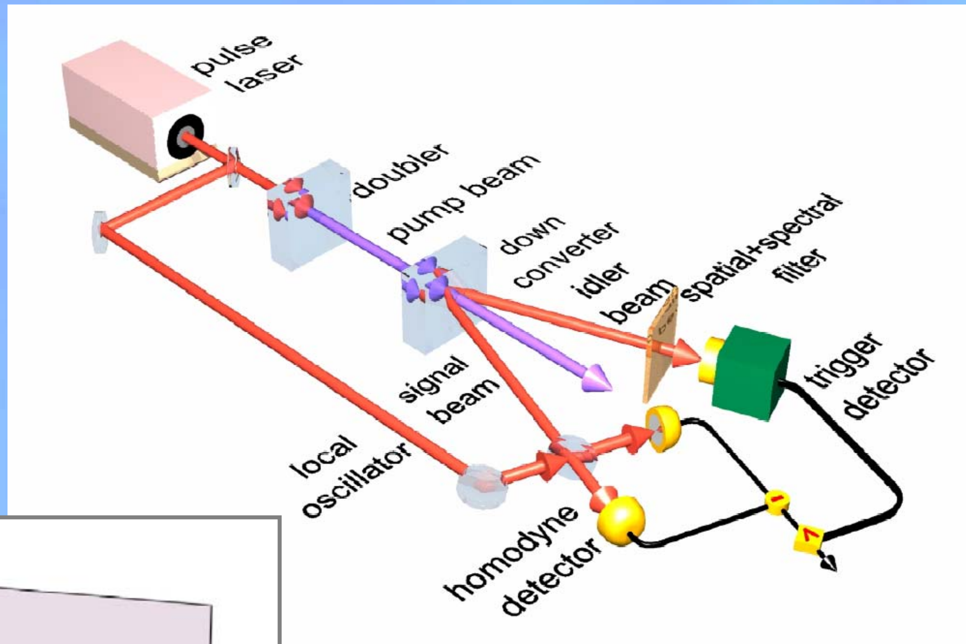
Localised one-photon Fock state

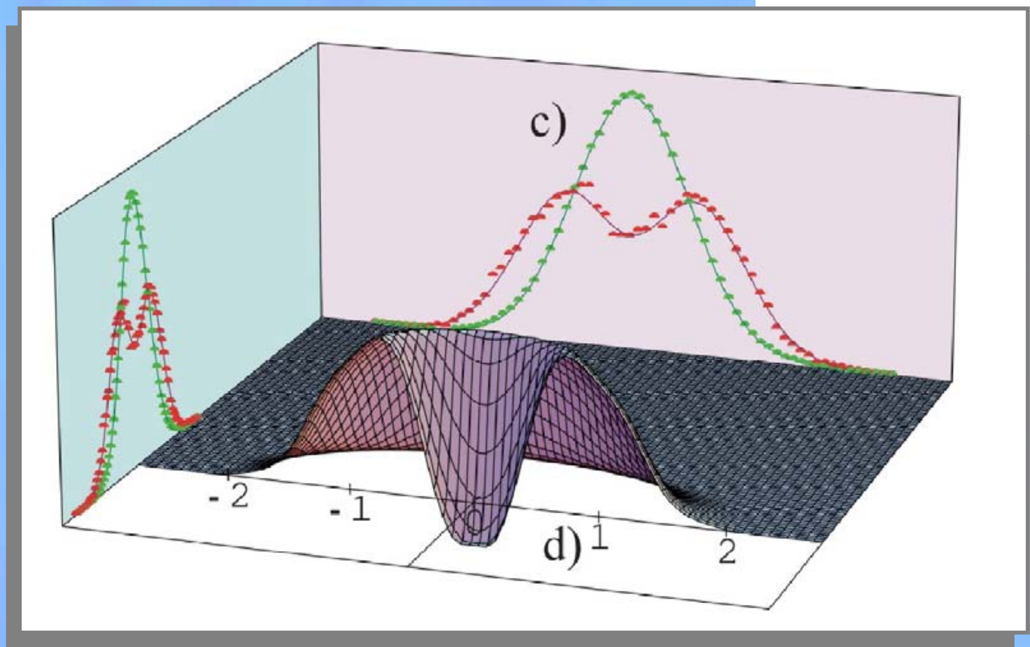


C. K. Hong and L. Mandel,
Phys. Rev. Lett. **56**, 58 (1986).



Homodyning Fock state

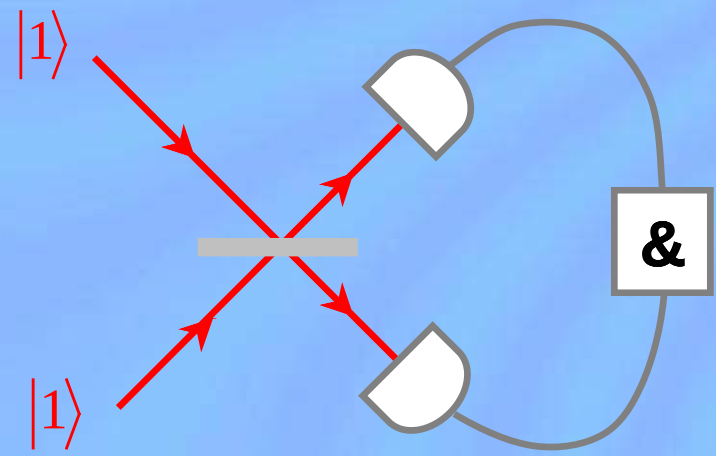




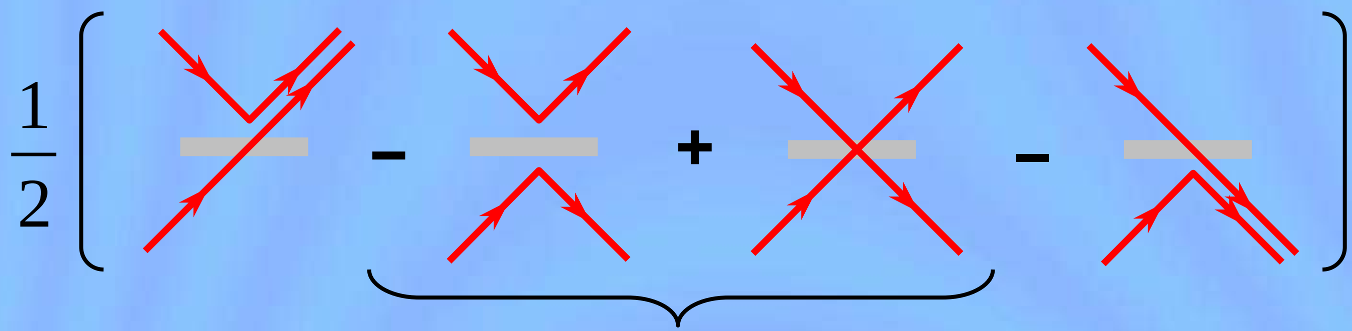
A. I. Lvovsky, H. Hansen, T. Aichele, O. Benson, J. Mlynek, and S. Schiller, Phys. Rev. Lett. **87**, 050402 (2001)

Two-photon interference I

Two-photon interference on a balanced beam splitter:



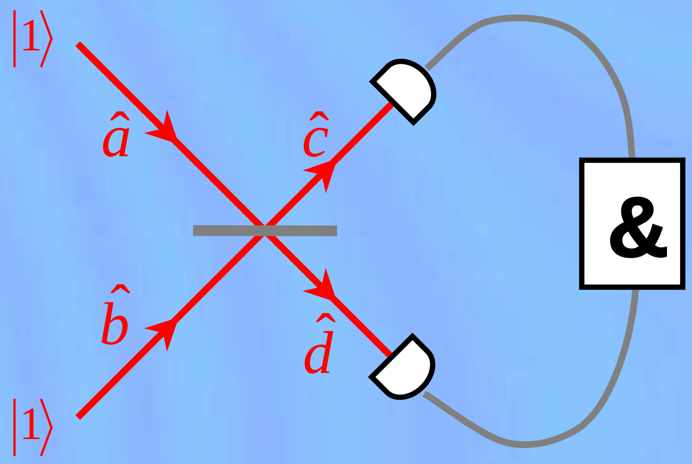
We have four probability amplitudes:



These amplitudes cancel when photons are indistiguishable!

For distinguishable photons all four amplitudes contribute equally.

Two-photon interference II



Incoming modes representation:

$$|1_a 1_b\rangle = \hat{a}^\dagger \hat{b}^\dagger |\text{vac}\rangle$$

Mode transformation:

$$\hat{a} = \frac{\hat{c}+\hat{d}}{\sqrt{2}} \qquad \hat{b} = \frac{\hat{c}-\hat{d}}{\sqrt{2}}$$

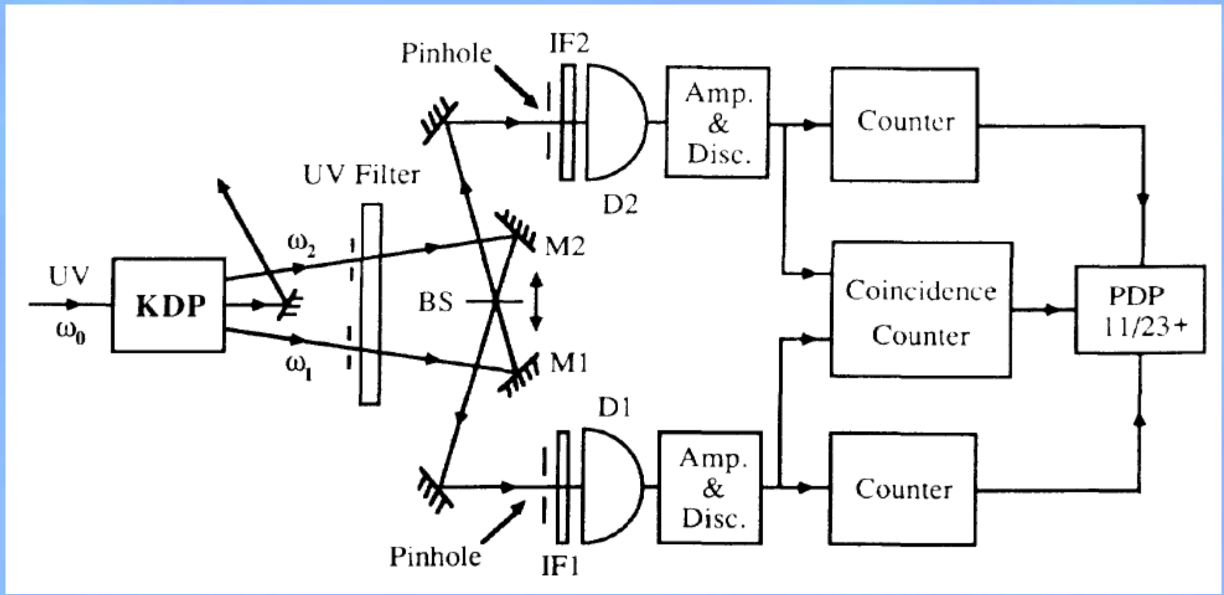
Outgoing modes representation:

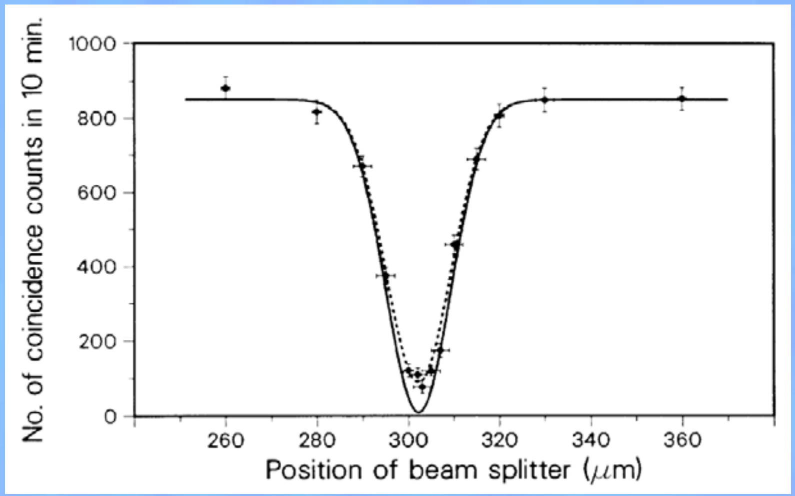
$$|1_a 1_b\rangle = \frac{1}{2}\left(\hat{c}^\dagger + \hat{d}^\dagger\right)\left(\hat{c}^\dagger - \hat{d}^\dagger\right)|\text{vac}\rangle = \frac{1}{2}\left[\left(\hat{c}^\dagger\right)^2 - \hat{c}^\dagger \hat{d}^\dagger + \hat{d}^\dagger \hat{c}^\dagger - \left(\hat{d}^\dagger\right)^2\right]|\text{vac}\rangle$$

$$= \frac{1}{2}\left[\left(\hat{c}^\dagger\right)^2 - \left(\hat{d}^\dagger\right)^2\right]|\text{vac}\rangle = \frac{1}{\sqrt{2}}\left(|2_c 0_d\rangle - |0_c 2_d\rangle\right)$$

Hong-Ou-Mandel interferometer

C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987)

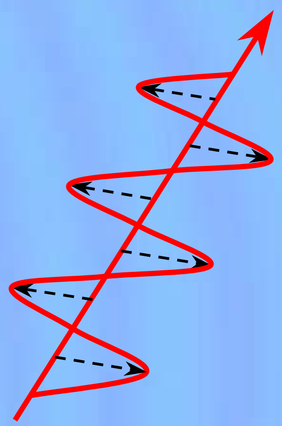
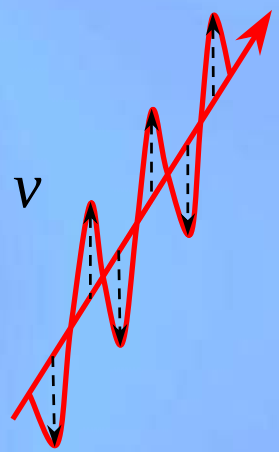
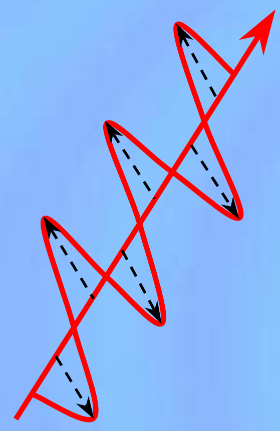




Polarization qubit

Two modes, orthogonally polarized,
but otherwise indistinguishable:

General superposition:

u  $+$ v  $=$ 

$$|\leftrightarrow\rangle = a^{\dagger}_{\leftrightarrow}|\text{vac}\rangle \qquad |\updownarrow\rangle = a^{\dagger}_{\updownarrow}|\text{vac}\rangle$$

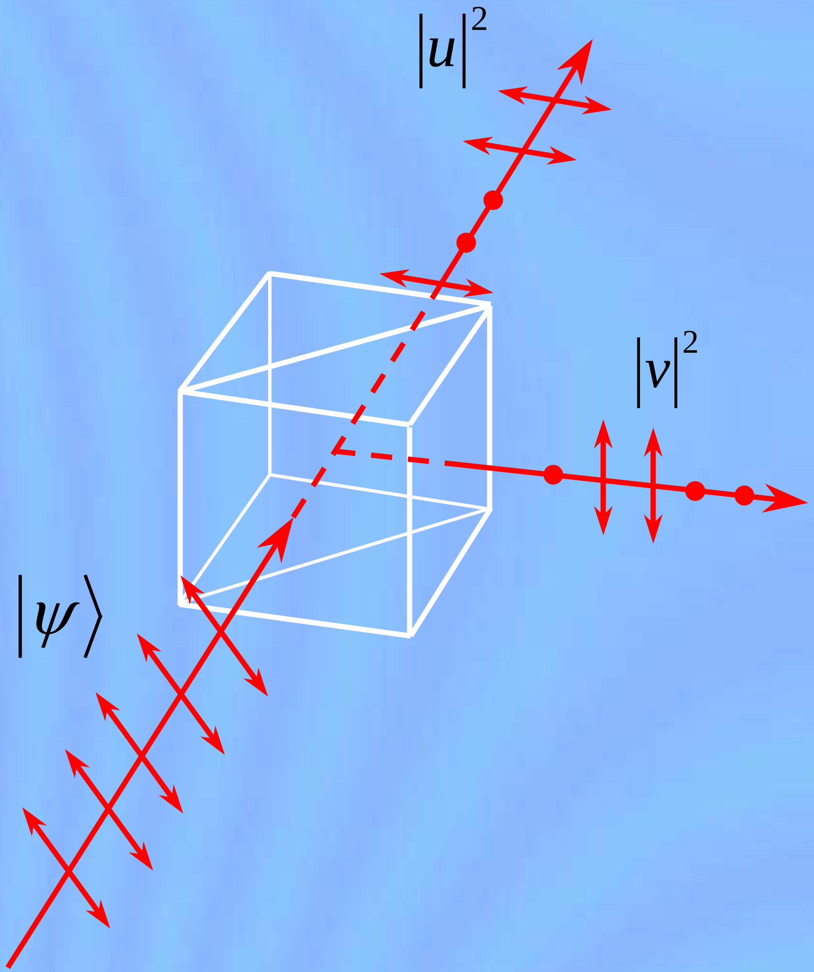
$$|\psi\rangle = u|\leftrightarrow\rangle + v|\updownarrow\rangle$$

$$= \left(u\hat{a}^{\dagger}_{\leftrightarrow} + v\hat{a}^{\dagger}_{\updownarrow}\right)|\text{vac}\rangle \equiv \begin{pmatrix} u \\ v \end{pmatrix}$$

Normalization requires that:

$$|u|^2 + |v|^2 = 1$$

Polarization measurement



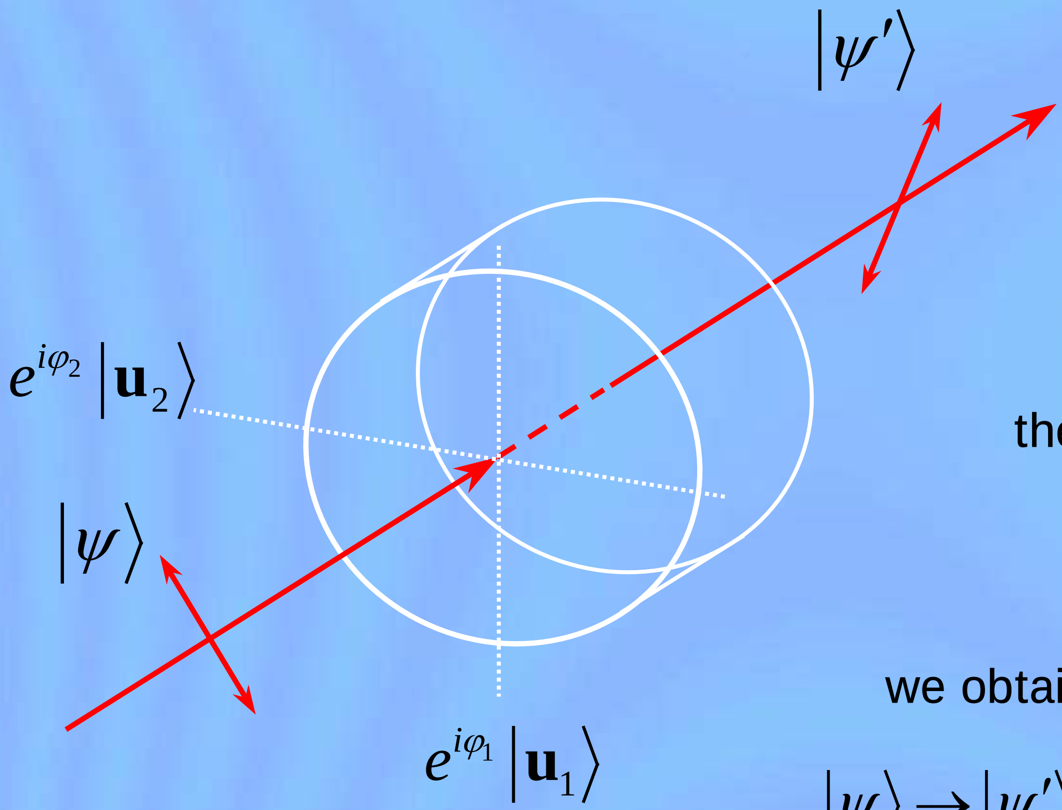
Probability of registering a photon

- transmitted: $|\langle\leftrightarrow|\psi\rangle|^2 = |u|^2$
- reflected: $|\langle\updownarrow|\psi\rangle|^2 = |v|^2$

A single measurement yields only partial information about the input state $|\psi\rangle$

Single-qubit gates

A wave-plate made of a birefringent material introduces phase delays φ_1 and φ_2 in directions $\mathbf{u}_1$ and $\mathbf{u}_2$:



If:

$$|\mathbf{u}_1\rangle \to e^{i\varphi_1}|\mathbf{u}_1\rangle$$

$$|\mathbf{u}_2\rangle \to e^{i\varphi_2}|\mathbf{u}_2\rangle$$

then for a general state

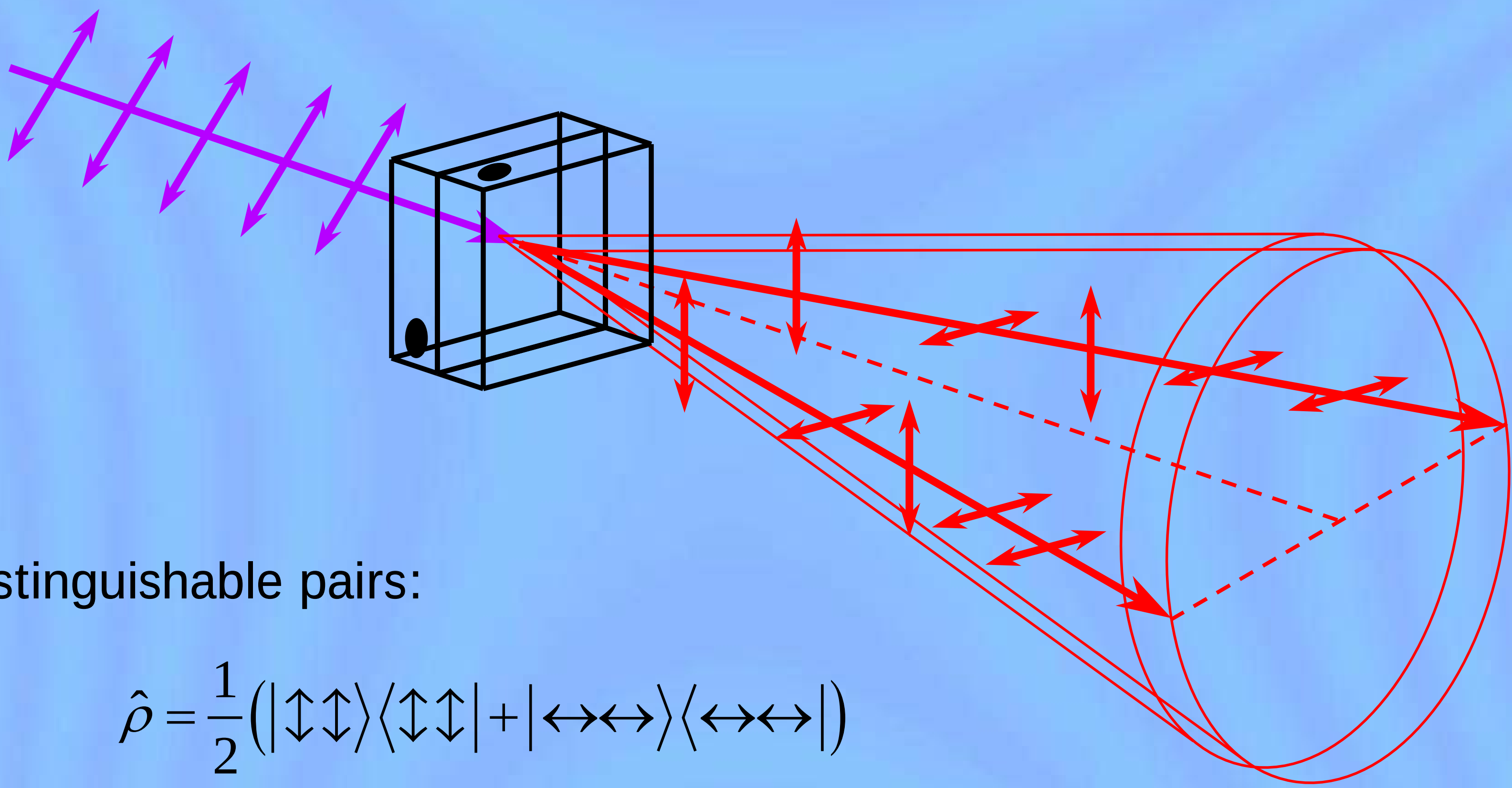
$$|\psi\rangle = \psi_1|\mathbf{u}_1\rangle + \psi_2|\mathbf{u}_2\rangle$$

we obtain:

$$|\psi\rangle \to |\psi'\rangle = e^{i\varphi_1}\psi_1|\mathbf{u}_1\rangle + e^{i\varphi_2}\psi_2|\mathbf{u}_2\rangle$$

This is a unitary transformation: $\hat{U} = e^{i\varphi_1}|\mathbf{u}_1\rangle\langle\mathbf{u}_1| + e^{i\varphi_2}|\mathbf{u}_2\rangle\langle\mathbf{u}_2|$

Two equally pumped crystals



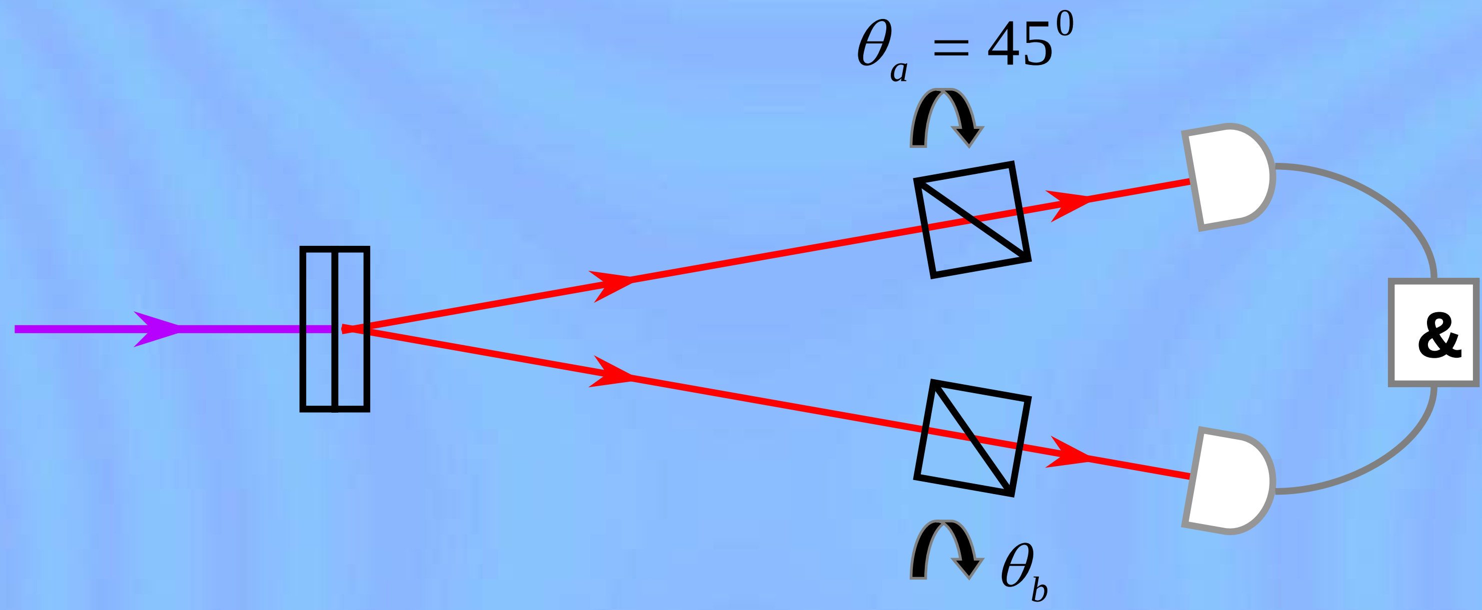
Distinguishable pairs:

$$\hat{\rho} = \frac{1}{2}\left(|\updownarrow\updownarrow\rangle\langle\updownarrow\updownarrow| + |\leftrightarrow\leftrightarrow\rangle\langle\leftrightarrow\leftrightarrow| \right)$$

Indistinguishable pairs (later we will set $\phi = 0$):

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\updownarrow\updownarrow\rangle - e^{i\phi}|\leftrightarrow\leftrightarrow\rangle \right)$$

Verifying entanglement



Coincidence probability for distinguishable pairs:

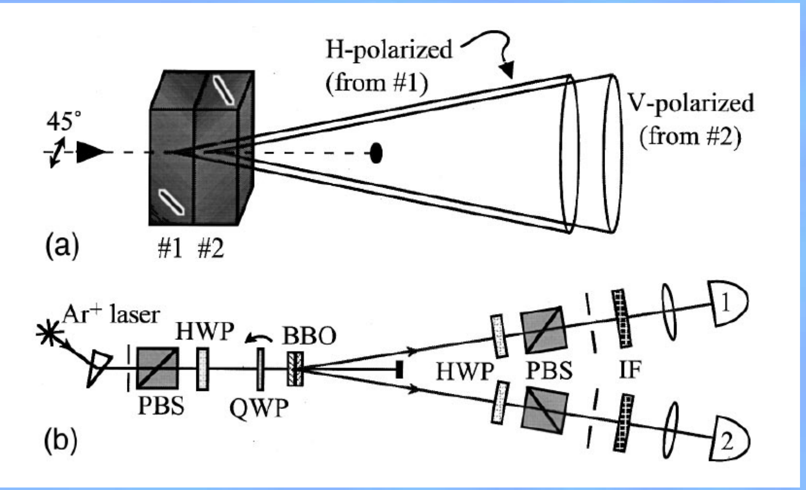
$$p\left(\theta_b \mid \theta_a = 45^0\right) = \frac{1}{2}$$

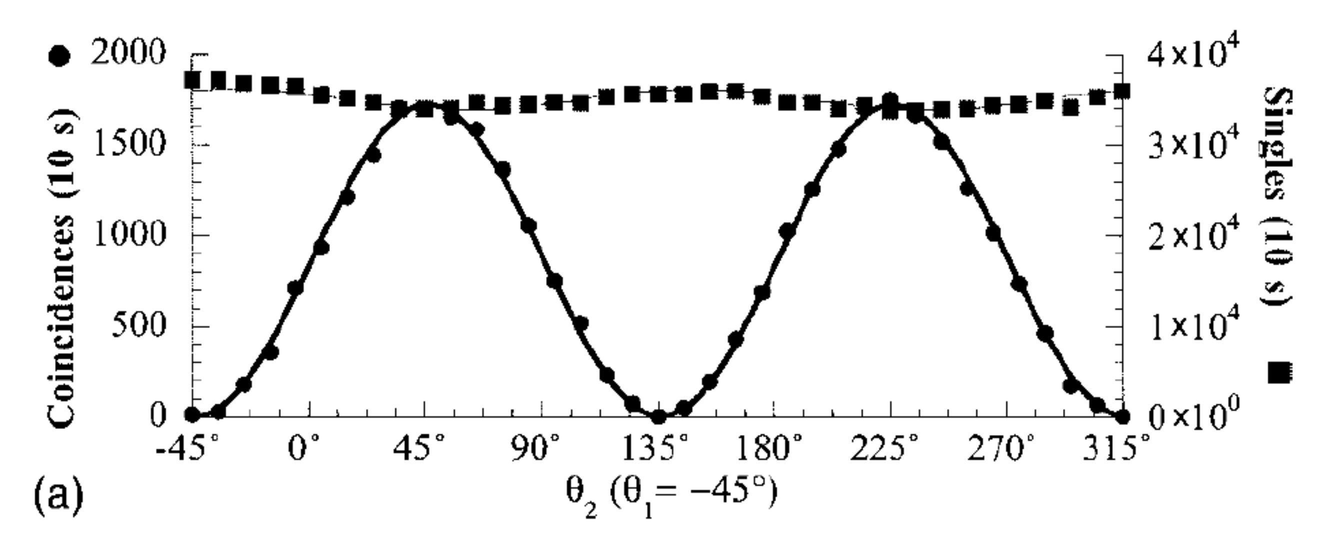
Coincidence probability for indistinguishable pairs:

$$p\left(\theta_b \mid \theta_a\right) = \cos^2\left(\theta_a + \theta_b\right)$$

Down-conversion source

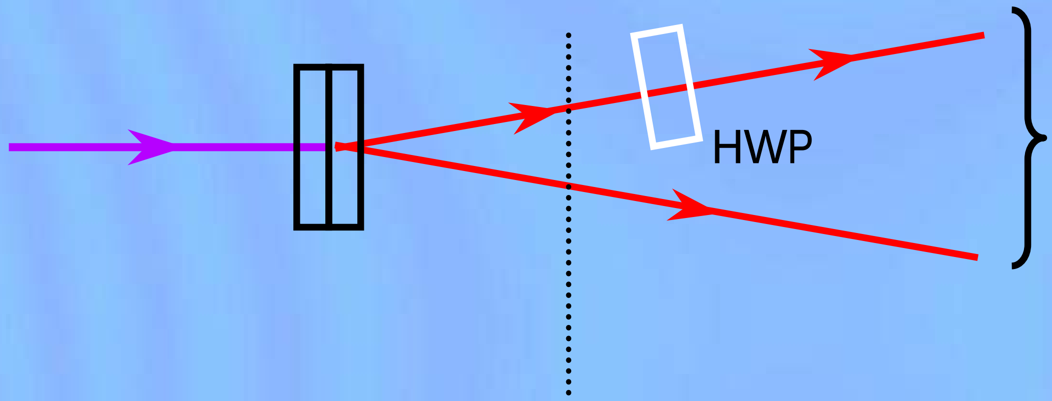
P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, Phys. Rev. A **60**, R773 (1999)





Singlet state

$$|\updownarrow\rangle \Leftrightarrow |\leftrightarrow\rangle$$



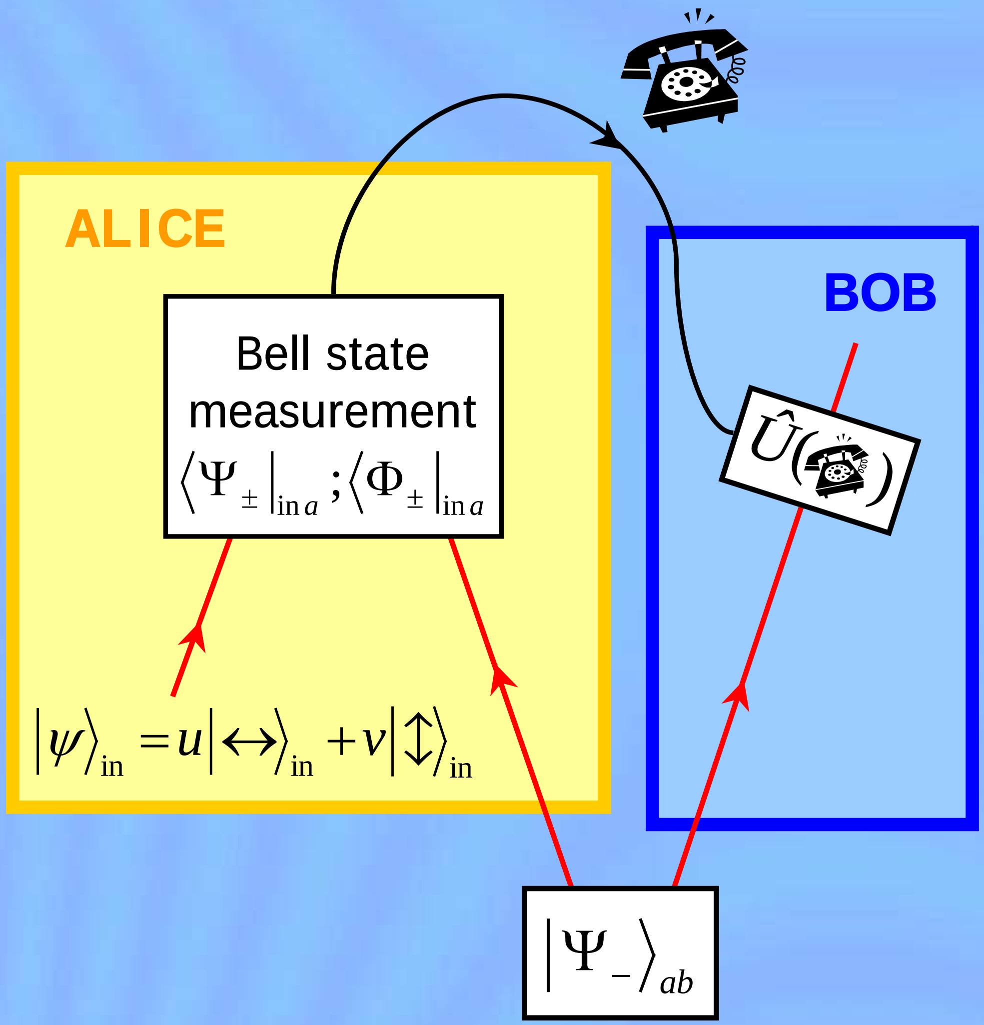
$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\updownarrow\rangle - |\updownarrow\leftrightarrow\rangle\right)$$

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\updownarrow\updownarrow\rangle - |\leftrightarrow\leftrightarrow\rangle\right)$$

Singlet state looks the same in *any* orthonormal basis:

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\updownarrow\rangle - |\updownarrow\leftrightarrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|\nearrow\nwarrow\rangle - |\nwarrow\nearrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|\circlearrowright\rangle|\circlearrowleft\rangle - |\circlearrowleft\rangle|\circlearrowright\rangle\right)$$

Qubit teleportation



Bell state basis:

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle|\updownarrow\rangle \pm |\updownarrow\rangle|\leftrightarrow\rangle\right)$$

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle|\leftrightarrow\rangle \pm |\updownarrow\rangle|\updownarrow\rangle\right)$$

Complete wave function:

$$\begin{aligned}
&\left(u|\leftrightarrow\rangle_{\text{in}} + v|\updownarrow\rangle_{\text{in}}\right)|\Phi_+\rangle_{ab} \\
&= \frac{1}{2}\Big[-|\Psi_-\rangle_{\text{in}\,a}\left(u|\leftrightarrow\rangle_b + v|\updownarrow\rangle_b\right) \\
&\quad + |\Psi_+\rangle_{\text{in}\,a}\left(-u|\leftrightarrow\rangle_b + v|\updownarrow\rangle_b\right) \\
&\quad + |\Phi_-\rangle_{\text{in}\,a}\left(u|\updownarrow\rangle_b + v|\leftrightarrow\rangle_b\right) \\
&\quad + |\Phi_+\rangle_{\text{in}\,a}\left(u|\updownarrow\rangle_b - v|\leftrightarrow\rangle_b\right)\Big]
\end{aligned}$$

C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993)

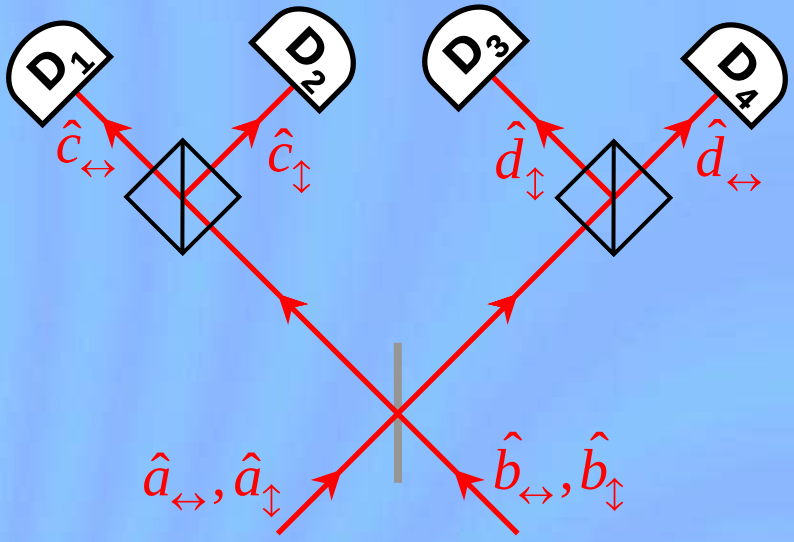
Bell state measurement

Coincidence on $\mathbf{D_1}$ and $\mathbf{D_3}$:

$$\langle \text{vac}|\hat{c}_{\leftrightarrow}\hat{d}_{\updownarrow} = \frac{1}{2}\langle \text{vac}|\left(\hat{a}_{\leftrightarrow}+\hat{b}_{\leftrightarrow}\right)\left(\hat{a}_{\updownarrow}-\hat{b}_{\updownarrow}\right)$$

$$= \frac{1}{2}\langle \text{vac}|\left(\hat{a}_{\leftrightarrow}\hat{a}_{\updownarrow}-\hat{b}_{\leftrightarrow}\hat{b}_{\updownarrow}\right) + \frac{1}{2}\langle \text{vac}|\left(\hat{a}_{\updownarrow}\hat{b}_{\leftrightarrow}-\hat{a}_{\leftrightarrow}\hat{b}_{\updownarrow}\right)$$

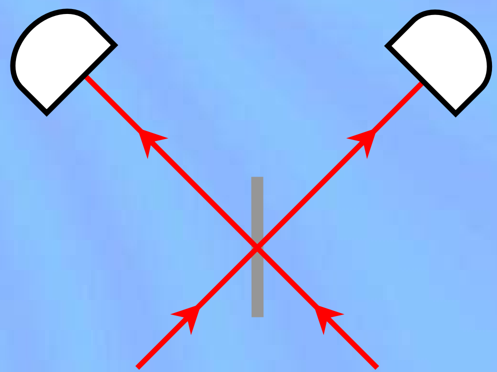
$$= \frac{1}{2}\left(\langle \updownarrow\leftrightarrow| - \langle \leftrightarrow\updownarrow|\right)$$



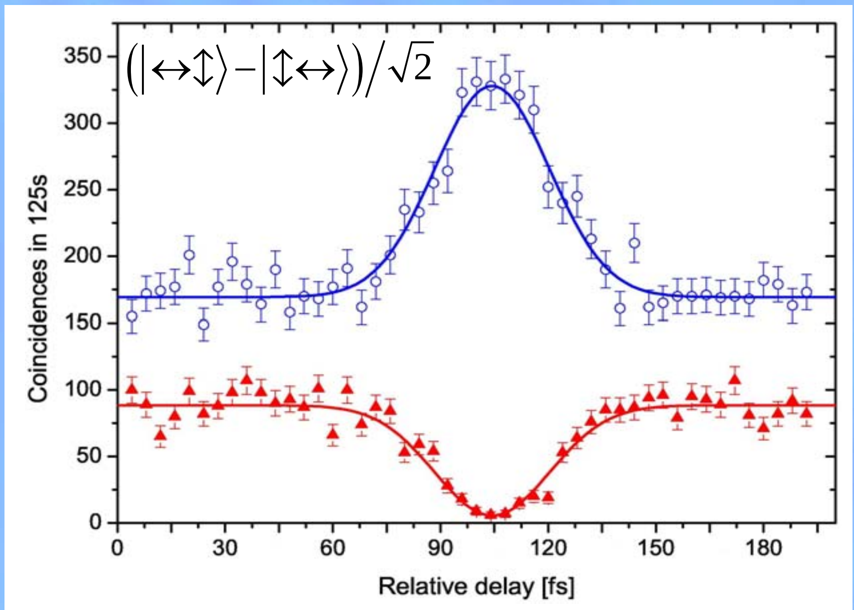
$\mathbf{D_1}$ & $\mathbf{D_3}$ or $\mathbf{D_2}$ & $\mathbf{D_4}$	$\frac{1}{\sqrt{2}}\left(\langle \updownarrow\leftrightarrow| - \langle \leftrightarrow\updownarrow|\right)$
$\mathbf{D_1}$ & $\mathbf{D_2}$ or $\mathbf{D_3}$ & $\mathbf{D_4}$	$\frac{1}{\sqrt{2}}\left(\langle \updownarrow\leftrightarrow| + \langle \leftrightarrow\updownarrow|\right)$

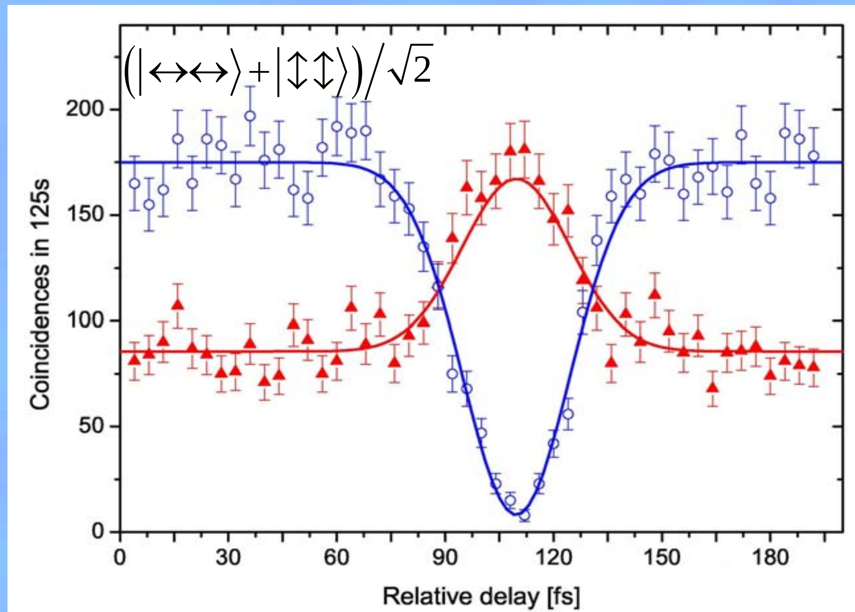
S. L. Braunstein and A. Mann, Phys. Rev. A **51**, R1727 (1995)

Singlet state vs. all the rest



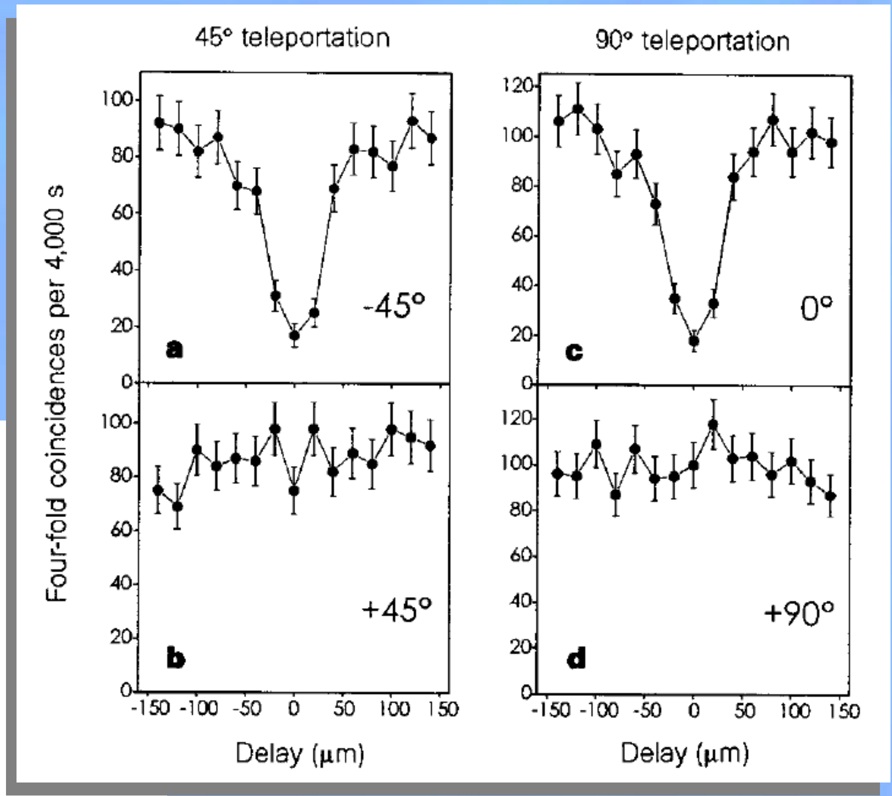
○	**1 & 1**	$\lvert\Psi_-\rangle\langle\Psi_-\rvert$
▲	**2 & 0** or **0 & 2**	$\lvert\Phi_-\rangle\langle\Phi_-\rvert+\lvert\Phi_+\rangle\langle\Phi_+\rvert+\lvert\Psi_+\rangle\langle\Psi_+\rvert$

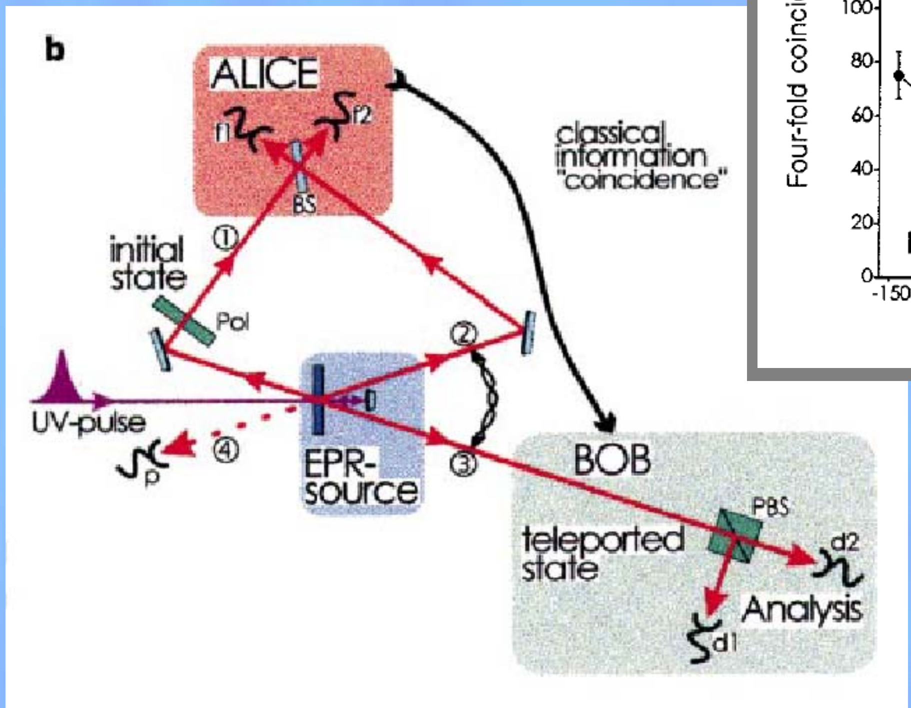




K. Banaszek, A. Dragan, W. Wasilewski, and C. Radzewicz, Phys. Rev. Lett. **92**, 257901 (2004)

Experimental teleportation





D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature **390**, 575 (1997)