Experimental Tests of Bell's Inequality - Lecture 1

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Lecture 1

1. The EPR argument- Bohm's version
2. Hidden Variables?
3. Locality and Bell's inequality
4. Bell's theorem
5. Relation to the Kochen-Specker and GHZ theorems

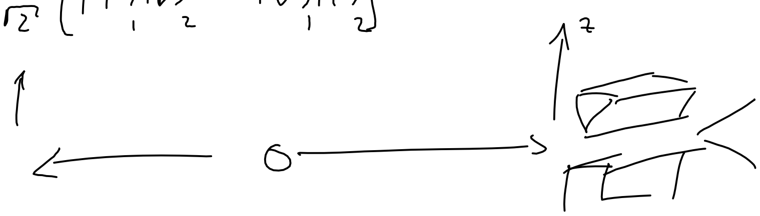
Lecture 2

1. Connection to Experiment
2. Early Experiments
3. Experimental Problems
4. Loopholes / Supplementary assumptions
5. The CH inequality
6. Aspect's experiments
7. My experiment
8. Trapped Ions and the efficiency loophole
9. Towards closing all the loopholes

The EPR-Bohm argument

Two spin 1/2 particles in singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

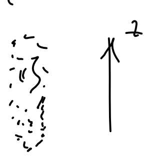


Stern Gerlach

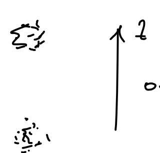
- Each time we measure 1 up → 2 down
  1 down → 2 up

- Whether up/down is unpredictable

- Classical spin $\overset{SG}{\Longrightarrow}$



Quantum $\Longrightarrow$



or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

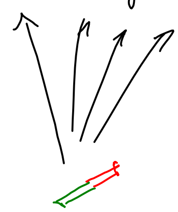
EPR: • By observing particle 1 we can choose to predict some property of particle 2 with certainty

- Because there can't be any spooky-at-a-distance (locality) we don't disturb particle 2, therefore we can choose what is real for particle 2 (reality)
- Therefore these properties must exist, i.e. QM is incomplete (completeness)

  ⇒ more complete description should be possible (hidden variables)

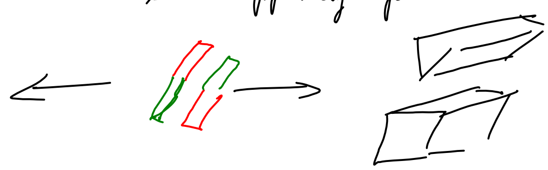
Example (HV): The force on a magnet in field produced

$F \cos\Theta$



"Quantize" $\quad \dfrac{F\cos\Theta}{|\cos\Theta|} = \pm |F|$

1) "Singlet" = pair of these magnets with random orientation (hidden variable) but oppositely poled



Because the two magnets share the precise axis they will produce perfectly anticorrelated results for parallel spin measurements.

2) Now, let's measure in different directions, say z and x

$z, x \Rightarrow$ no correlation

arbitrary SG angles a, b

$$p_{\uparrow\uparrow}(a,b) = p_{\downarrow\downarrow}(a,b) = \frac{1}{2\pi}|a-b|$$

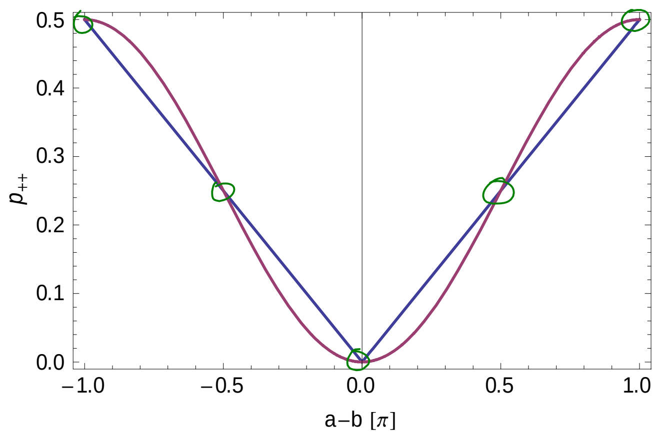
$$p_{\uparrow\downarrow}(a,b) = p_{\downarrow\uparrow}(a,b) = \frac{1}{2} - \frac{1}{2\pi}|a-b|$$

But quantum mechanically

$$p_{\uparrow\uparrow}(a,b) = \quad \dots \quad = \frac{1}{2}\sin^2(a-b)$$

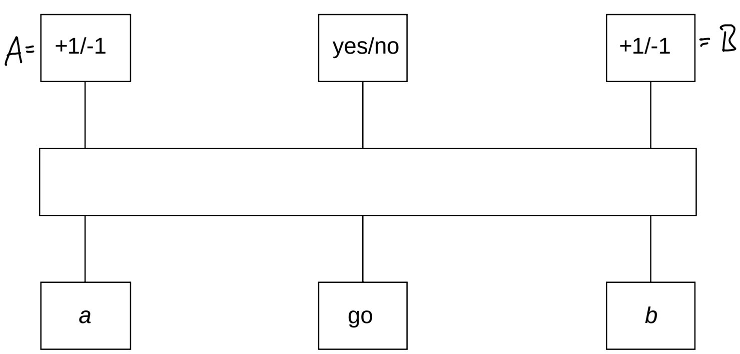
$$p_{\uparrow\downarrow}(a,b) = \quad \dots \quad = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$$

$$p_{\uparrow\downarrow}(a,b) = \quad \dots \quad = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$$



naive model reproduces these points at $0, \pm\frac{\pi}{2}, \pm\pi$

? Could we do better? $\Rightarrow$ no! Bell's theorem



Bell's inequality

1) $\exists \; p(A,B\,|\,a,b,\lambda)$ $\quad \lambda$… hidden variables $\quad A,B \in \{\pm 1\}$

$\quad A(a,\lambda) \;,\; B(b,\lambda) \quad \in \{\pm 1\}$

2) $p(A,B\,|\,a,b,\lambda) = p(A\,|\,a,\lambda)\cdot p(B\,|\,b,\lambda)$ $\quad$ Locality

$\Leftrightarrow$ parameter independence + outcome independence

3) Freedom $\quad a,b$ can be chosen independently of λ

$$\langle AB\rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

$\in [-1,1]$

$$p_{++}^{\lambda}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda) - p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda) - p_-(b,\lambda)]}_{\in[-1,1]}$$

Lemma: for $\rho,\rho',r,r' \in [-1,1]$ $\quad S := \rho r + \rho r' + \rho' r - \rho' r' \in [-2,2]$

Proof: 1) Because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $\rho,\rho',r,r' \in \{\pm 1\}$

$S_{EXT} \in [-4,4]$

2) $S_{EXT} = \underbrace{\underbrace{(\rho+\rho')}_{\in\{-2,0,2\}}\underbrace{(r+r')}_{\in\{-2,0,2\}}}_{\in\{-4,0,4\}} - \underbrace{2\rho' r'}_{\in\{\pm 2\}} \Rightarrow \in \{\cancel{-6}, -2, 2, \cancel{6}\}$ $\quad\square$

$$S^{\lambda} := E^{\lambda}(a,b) + E^{\lambda}(a,b') + E^{\lambda}(a',b) - E^{\lambda}(a',b')$$

with $\rho = p_+(a,\lambda) - p_-(a,\lambda)$

$\rho' = p_+(a',\lambda) - p_-(a',\lambda) \qquad \Rightarrow -2 \le S^{\lambda} \le 2$

$\vdots$

$S^S := \int S^{\lambda}\rho(\lambda)\,d\lambda \qquad \rho \ldots \qquad \int \rho(\lambda)\,d\lambda = 1$

$E^S := \int E^{\lambda}\rho(\lambda)\,d\lambda$

$\Rightarrow \boxed{|S^S| \le 2}$ **Bell's inequality**

The naive model: $\quad p_{++}(a,b) = \frac{1}{2\pi}|a-b|$

$$E(a,b) = \frac{1}{\pi}|a-b| - 1 + \frac{1}{\pi}|a-b| = \frac{2}{\pi}|a-b| - 1$$

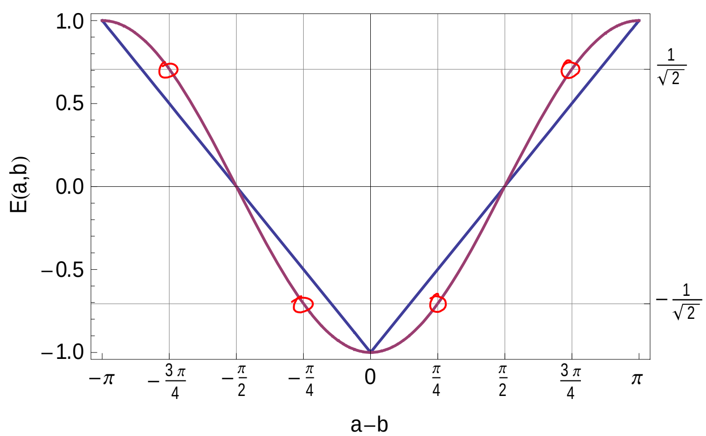
Bell's theorem

quantum mechanically $\quad |\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right]$

$$P_{++}(a,b) = \left|\langle\uparrow_a|\langle\uparrow_b|\psi\rangle\right|^2 \qquad \begin{aligned}|\uparrow_a\rangle &= \cos\frac{a}{2}|\uparrow_z\rangle + \sin\frac{a}{2}|\uparrow_z\rangle \\ |\downarrow_a\rangle &= \qquad\quad -\end{aligned}$$

$$P_{++}(a,b) = \frac{1}{2}\left|\cos\frac{a}{2}\sin\frac{b}{2} - \sin\frac{a}{2}\cos\frac{b}{2}\right| = \frac{1}{2}\sin^2\frac{a-b}{2}$$

$$\Rightarrow E^{qm}(a,b) = \ldots = -\cos(a-b)$$



choose $\quad a = 0, \quad a' = \frac{\pi}{2}, \quad b = \frac{\pi}{4}, \quad b' = \frac{3\pi}{4}$

$$|S^{qm}| = \left|-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(+\frac{1}{\sqrt{2}}\right)\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2} > 2$$

- This is the maximal violation of Bell's inequality
- All maximally entangled states give this value
- Separable states don't violate BI

Relation to the Kochen-Specker and GHZ theorems