Lecture 5: Modern Perspectives on Interpretation

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Foundations and Interpretations of Quantum Theory

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1. Some Contemporary Interpretations
2. What does it mean to interpret a theory?
   - Operational Interpretation
   - Minimal Statistical Interpretation
3. Ontological Interpretations
   - Interpretation of Newtonian/Hamiltonian mechanics
   - Are Quantum States Ontic or Epistemic?
4. Ontological Models for Quantum Theory
   - Kochen-Specker Model
5. Constraints on Hidden Variable Models
6. Non-locality

Some Contemporary Interpretations

The measurement problem of the orthodox interpretation and Bohr's insistence upon dividing the world into a quantum part (observed) and a classical part (measurer) has motivated confusion and unease amongst students of quantum mechanics for generations.

- This despair has motivated a wide-variety of mutually incompatible "solutions to the measurement problem."
- One of the prominent themes of contemporary interpretations is to overcome the measurement problem.
- The usual idea is to develop an interpretation of quantum mechanics which does not need to distinguish 'measurement' from all other naturally occurring process and for which there is no special role for 'the observer' or 'measurement'.

Some Contemporary Interpretations

Many-Worlds Interpretation:
The *many-worlds interpretation* (sometimes branded as the "many-words" interpretation) grew out of the relative-state interpretation developed by Everett (1957).

- The basic idea is to propose that reality itself divides into alternate but equally valid branches of reality whenever the state vector does so (mathematically) under unitary evolution.
- In this way the the projection postulate is not necessary, because from the perspective of "each version of the observer", ie, in each branch, their is a unique measurement outcome associated with that branch of reality.
- This interpretation was presented for this course in 2005 by D. Wallace and will be presented this March by Lev Vaidman.

Some Contemporary Interpretations

Decoherence-based Interpretations:
In many contemporary interpretations the effects of decoherence play a pivotal role in defining the ontology (or, more precisely, ontologies), and in particular the "preferred basis":

- One example is the *existential interpretation* advocated by W. Zurek (1993), which trades on features of the many-worlds interpretation.
- Another example is the *decoherent/consistent histories* interpretations, developed by R. Griffiths (1984) and extended by Gell-Mann and Hartle (1990), which was presented for this course in 2005 by R. Griffiths.

Some Contemporary Interpretations

Dynamical Collapse Models:
Dynamical collapse interpretations specify a physical/dynamical mechanism which produces an actual collapse of the wavefunction - typically by adding a non-linear term to the Schrodinger equation.

- Strictly speaking these interpretations are actual modifications of quantum theory and not just interpretations.
- As a result they are in principle falsifiable although it is not evident how to distinguish the proposed collapse mechanism from the ubiquitous effects of decoherence.

Some Contemporary Interpretations

Dynamical Collapse Models (continued):

- Their approach is to change quantum theory (which works just fine) in order to make the orthodox interpretation self-consistent, rather than accept quantum theory and recognize that the orthodox interpretation is just a bad interpretation.
- P. Pearle described his version, called continuous spontaneous localization, in the 2005 version of the course.
- Another well-known approach is the GRW scheme.
- Roger Penrose also has a proposed a dynamical collapse theory which ties in gravitational effects.

Some Contemporary Interpretations

The Statistical Interpretation:

The statistical interpretation, developed by Ballentine (1970), is modeled after Einstein's views.

- The defining feature is a rejection of the assumption that quantum states give a complete account of the ontology for an individual system and emphasize that the quantum state gives a description of statistical ensembles of similarly prepared systems.
- In this way the interpretation makes no ontological commitments and hence has no measurement problem.
- Implicitly the view presupposes that the actual ontology of the world is described by additional "hidden" variables, our incomplete knowledge of which gives rise to the statistics of quantum theory, although the specific details of the hidden variables are left to be determined.
- This interpretation was presented by Leslie Ballentine in the 2005 version of this course.

Some Contemporary Interpretations

Bohmian mechanics:
By far the most prominent hidden variable interpretation is the *deBroglie-Bohm pilot wave theory* (1927/1952), in which particles always have a definite positions and momenta - these are "hidden variable" because they are not manifest in the usual mathematical formulation of quantum mechanics.

- The hidden variables evolve deterministically and produce definite measurement outcomes, so there is no measurement problem nor any special role for the observer.
- The quantum state describes the probability density over these "hidden variables" and also generates an additional "quantum potential" which provides an additional force that guides the particles away from the usual classical trajectories associated with the system Hamiltonian.

Some Contemporary Interpretations

Bohmian mechanics (continued):

- The theory reproduces the predictions of quantum mechanics exactly.
- The interpretation is explicitly non-local (although it of course does not allow faster-than-light signalling) and also contextual.
- The Bohmian interpretation was presented by Shelly Goldstein in 2005 and will be presented this year by Roderich Tumulka.

Operational Bridge Principles

What does it mean to interpret a theory?
The minimum that is required is a set of “operational bridge principles” that connect the mathematical elements of the theory to experimentally verifiable phenomena.

- These operational bridge principles are rules that enable the theory to predict experimental outcomes.
- This level of interpretation fulfills the practical role of a physical theory as a means of predicting and controlling physical systems.

Minimal Statistical Interpretation

In quantum theory, we need the following operational bridge principles:

- We need bridge principles that associate observable properties with appropriate self-adjoint operators, and associate preparation procedures with appropriate state operators.
  - This will require certain "background knowledge", such as consideration of space-time symmetries, and calibration procedures.
- The most prominent bridge principle is Born's rule,

$$p_k = \mathrm{Tr}(\rho P_k),$$

  where P_k is a projector in the spectral decomposition of some observable, and ρ is a state operator.
  - Born's rule assigns probabilities (relative frequencies !) to the set of observable outcomes (indexed by k).

Minimal Statistical Interpretation

The minimal statistical interpretation

Quantum mechanics makes statistical predictions about what outcomes may be found when specified measurement procedures are performed upon specified preparation procedures.

- This an "ontology-free" interpretation of quantum mechanics.
- Under this view quantum theory tells us nothing about what properties or facts obtain in the world at any particular time.
- Because we make no ontological commitments, there is no measurement problem.
- Note that for simplicity we are ignoring transformation (as they can always be absorbed in the specification of preparation).

Minimal Statistical Interpretation

- The minimal statistical interpretation is just ordinary textbook quantum mechanics (minus the usual lip-service to Bohr, Dirac and von Neumann)
  - That is, provided one understands that the projection postulate is an update rule that is applied only *after* the outcome is observed (in order to describe the conditional state after measurement).
- From a pragmatic view, one can be content with this minimal interpretation because it is sufficient to apply the theory to describe physical phenomena.

Minimal Statistical Interpretation

- This is the minimal aspect of interpreting quantum theory which scientists must agree upon.
- Indeed this is precisely the point of the "shut up and calculate" school of thought, which argues that no further interpretation is needed.

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Stephen Hawking

"When I hear about Schrodinger's cat, I reach for my gun."

Minimal Statistical Interpretation

The minimal statistical interpretation is the operational/instrumentalist view advocated by Peres.

> "In a strict sense, quantum theory is a set of rules allowing computation of probabilities for the outcomes of tests which follow specified preparations."

A. Peres (1995)

- Under the minimal statistical interpretation, the theory is still an input-output theory, ie, it has no more ontological significance than the rules in an instruction manual.

Minimal Statistical Interpretation

I take this view to differ from the statistical interpretation proposed by Einstein and Ballentine because they further argued that the correctness of this instrumentalist interpretation suggested a more complete theory (with a proper ontology) should be possible.

- Indeed the whole point of Ballentine/Einstein was to just convince researchers to stop thinking about pure quantum states as ontic elements of reality, ie, to drop the literal realism of the orthodox interpretation, and thereby disabuse themselves of the paradoxes and conceptual confusions that plague that interpretation of quantum theory.

> "Ordinary quantum mechanics (as far as I know) is just fine FOR ALL PRACTICAL PURPOSES [FAPP]"

John Bell

- **But what is really going on?**

In spite of the practical solution offered by operationalism, it is nonetheless interesting to ask: what can we say about reality that is consistent with quantum theory.

- That is, we demand a set of *ontological bridge principles* (from the Greek *ontos*, meaning 'to be').
- Ontological bridge principles draw relations between the elements of the mathematical formalism and elements of reality, ie, facts about the world.

Ontological Bridge Principles

- These principles are essential for the explanatory role of the physical theory, that is, to provide some story about the nature of the physical world.
- The ontological bridge principles can also give intuition about how systems behave and about what results to expect when direct calculation is infeasible.
- Moreover, the answers can hopefully give insight into how to unify our physical theories (e.g., the problem of unifying quantum theory with gravity.)

Interpretation of Newtonian/Hamiltonian mechanics

A good example of an ontological theory is Hamiltonian/Newtonian mechanics.

- The objective properties (q, p) may be assigned to a system - these are the ontic state.
- These properties evolve deterministically according to the canonical equations of motion.
- The state $(q(t), p(t))$ is "complete" in the sense that these values determine the physical facts of the world at any given time.
- Moreover, combined with the dynamical laws, these facts provide all the information that is needed to determine the physical state of the system at any time.

Interpretation of classical Liouville mechanics

A good example of a theory that does not admit an ontological interpretation is classical Liouville mechanics.

- Liouville mechanics is designed to describe classical systems about which we only have "incomplete information" of the ontic states, which is the practical situation.
- The "state" of the theory is a probability density over the classical ontic states:

$$\rho(q, p, t).$$

- The "state" evolved linearly and deterministically under the partial differential equation:

$$\frac{\partial \rho(q,p,t)}{\partial t} = \{H, \rho(q,p,t)\}.$$

Interpretation of classical Liouville mechanics

- The "states" of Liouville mechanics do not admit an ontological interpretation (except for the Dirac delta-function states).
- The "states" are only "epistemic" objects (from the Greek 'episteme', meaning knowledge).
- The states of Liouville mechanics reflect "incomplete information" rather than objective properties.
- The point is: the states of a theory need not be ontic states!

Are Quantum States Ontic or Epistemic?

- What are quantum states? Are they ontic or epistemic objects? Or both?
- If they are epistemic, then:
  - what are the (hidden) ontic variables?
  - what constraints might quantum theory impose on the possible hidden variables?
  - is a fully deterministic dynamical theory for the hidden variables possible?
- If they are ontic, then:
  - do they specify the complete ontology, or are their additional (hidden) ontic variables?
  - how can we overcome the measurement problem and/or specify when and how the physical collapse occurs?

Ontological Models for Quantum Theory

The idea of the ontological models framework (term coined by Rob Spekkens, I think) is to bring some order to ontological claims about quantum mechanics.

- Let any ontology associated with an interpretation of quantum theory comprise a set Λ, where each ontic state $\lambda \in \Lambda$ gives a complete specification of the physical properties of the system.
- The state λ is the real state of affairs, the physical configuration, or simply, the ontology.
- For example, for a classical Hamiltonian system of n particles in $3d$, the ontic space Λ is the phase space R^{3n}.

Ontological Models for Quantum Theory

We can elevate this ontology to a predictive framework (for an operational theory) if we associate any preparation P with a probability measure on Λ, and any measurement M with a conditional probability ξ, such that,

$$Pr(k) = \int_{\Lambda} \mu_P(\lambda)\xi_M(k|\lambda).$$

Ontological Models for Quantum Theory

We can express Liouville mechanics in this formalism:

- A preparation is a probability density $\rho(x, p, t)$
- A measurement that checks whether the particle lies in a phase space region $S \subset R^{3n}$ is given by a conditional probability $\xi_S(x, p)$ which satisfies
$$\xi_S(x, p) = 1$$
if $\{x, p\} \in S$ and
$$\xi_S(x, p) = 0$$
otherwise.
- Then letting $\lambda = (x, p)$ we have
$$Pr(\{x, p\} \in S) = \int_\Lambda \rho(\lambda, t)\xi_S(k|\lambda) = \int_S \rho(\lambda).$$
- For this kind of measurement the conditional probability is an *idempotent* indicator function, $\xi^2 = \xi$.

Ontological Models for Quantum Theory

If the ontological model is meant to reproduce the predictions of quantum theory then $P \to \rho$ and $M \to E_K$ and we demand:

$$Pr(k) = \int_\Lambda \mu_P(\lambda)\xi_M(k|\lambda) = \mathrm{Tr}(\rho E_k).$$

- There are many examples of ontological models that reproduce the predictions of quantum theory; a restricted version of Liouville mechanics is one of them (though only for the case $d = 2$) as we will see.

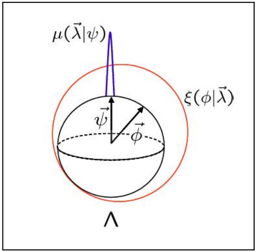
Ontological Models for Quantum Theory

A *hidden variable theory* as an ontological model where (at least one) pure quantum state is associated with a probability measure on the ontic space Λ that has support on more than one element of Λ, and hence the ontic state of affairs is not fully specified by the quantum state.

- An example of an ontological model that is not a hidden variables model is the Beltrametti-Bugjaski model.
- In this model the ontic space is just the projective Hilbert space itself: $\Lambda = CP^{d-1}$.

Ontological Models for Quantum Theory

- Pure states are constructed as delta-functions on this space $\mu(\lambda) = \delta(\lambda - |\psi\rangle\langle\psi|)$ and by convexity we can construct mixed states.
- Measurement operators are non-idempotent conditional probabilities: $\xi(k|\lambda) = \mathrm{Tr}(E_k \lambda)$



- Remark that this ontological model is just a formalization of the orthodox interpretation, where reality itself is assumed to be completely specified by the wavefunction.

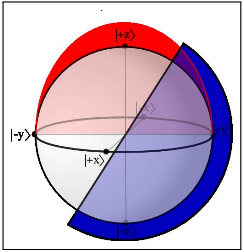
Hidden Variable Models for Quantum Theory

A simple example of a *bona fide* hidden variable model is the Kochen-Specker model (1967):

- The KS model reproduces the quantum predictions for arbitrary density operators and all proper POVMS on a 2-dimensional Hilbert space.
- The ontic space is $\Lambda = S^2$, as you would expect for a classical angular momentum of fixed length.
- The model can be described completely using classical Liouville mechanics.
- Hence a single qubit is completely classical.

Kochen-Specker Model

- Pure states are $\cos\theta$ distributed probability densities centered along the direction of S^2 corresponding to vector formed from the quantum expectation values $\vec{S} = \langle S_x \rangle, \langle S_y \rangle, \langle S_z \rangle)$, with support only on the hemisphere centered as above.



- Projective measurements are idempotent indicator functions with support on the hemisphere centered about the direction of the associated projector.
- By convexity we can construct measures for arbitrary density operators and proper POVMs.

Incompleteness and hidden variables:

- The possibility of hidden variables was considered and rejected by von Neumann, who produced an 'impossibility proof' based on three assumptions.
- First, the hidden variables assignments were required to completely specify experimental outcomes (i.e., to produce dispersion-free states) for all Hermitian operators;
- Second, the unique hidden value assignment to each operator was required to be one of the operator's eigenvalues;
- Third, for any Hermitian operator $\hat{C} = a\hat{A} + b\hat{B}$ defined by a linear combination of arbitrary (e.g., non-commuting) Hermitian operators, the hidden value assignment for $\hat{C}$ was required to satisfy the same linear combination of the hidden value assignments for the operators $\hat{A}$ and $\hat{B}$.

However, the third assumption is incompatible with the first and second assumptions.

- Consider the Pauli operator defined by,

$$\sigma_n = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_y).$$

- The eigenvalues of all three Pauli operators are ± 1, but clearly, the eigenvalues of σ_n can not be expressed as any of the linear combinations,

$$\frac{(\pm 1 \pm 1)}{\sqrt{2}}.$$

von Neumann's third assumption is generally considered unjustified (even "silly" - by Mermin (1993)), since it imposes constraints on the hidden value assignments for incompatible experimental arrangements. His assumption appears to be inspired by the fact that this relation holds for quantum mechanical expectation values ($\langle C \rangle = \langle A \rangle + \langle B \rangle$).

- Note that the KS model is a perfectly valid hidden variable model for d=2 quantum mechanics but it does not satisfy von Neumann's assumptions (because there is a difference between the value the hidden variable has and the discrete outcomes associated with the distinct measurements).

If we drop the third assumption then hidden variable models can be, and indeed have been, constructed - see Bell (1966) for the complete analysis and a simple example.

- The most celebrated example is the de Broglie-Bohm hidden variable model (1927/1952), which has an explicit non-local character.
- This explicit non-local character of the de Broglie-Bohm hidden variables and the crucial role of "locality" in the EPR-Bohm argument led Bell to ask: is non-locality a necessary feature of any hidden variable theory reproducing the predictions of quantum theory? He showed that the answer is yes (Bell (1964)).

Recall Einstein characterized ‘locality’ as follows (1949):

Einstein Locality

“The real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former.”

Bell's Theorem:

- Bell considered a restriction on the correlations that can be exhibited between two systems in the EPR-Bohm set-up allowing for the fact that the outcomes could be determined by an arbitrary class of (deterministic) hidden variables.

Consider two spatially separated spin systems each subjected to measurement along directions a and b respectively. The results of the measurements, denoted A and B, can depend on arbitrary parameters (hidden variables) collectively denoted λ, and can take on the values $|A| \leq 1$ and $|B| \leq 1$.

- The outcome can of course depend on the local setting, but, by *assuming Einstein locality*, is not allowed to depend on the setting of the distant instrument. Hence $A = A(a, \lambda)$ and $B = B(b, \lambda)$ are allowed but $A = A(a, b, \lambda)$ and $B = B(a, b, \lambda)$ are excluded by the locality assumption.

The uncontrolled parameters are governed by an arbitrary probability density $\rho(\lambda)$, where,

$$\rho(\lambda) \geq 0, \qquad \int \rho(\lambda) d\lambda = 1,$$

and hence we can define correlations of the form:

$$C(a,b) = \int A(a,\lambda)B(b,\lambda)\rho(\lambda)d\lambda$$

- Each detector is allowed to have two *independently selected* settings $\{a, a'\}$ and $\{b, b'\}$. From these assumptions we can derive Bell's inequality (see Bell 1964):

$$|C(a,b) - C(a,b')| + |C(a',b') + C(a',b)| \leq 2$$

A quantum mechanical system satisfying Bell's assumption consists of two spin-1/2 particles (or generic two-level systems) in the singlet state,

$$\psi = \frac{1}{\sqrt{2}} \left(|+\rangle_A \otimes |-\rangle_B \; - \; |-\rangle_A \otimes |+\rangle_B \right).$$

- Define the observable correlation as,

$$C(\mathbf{a}, \mathbf{b}) = (\mathbf{2}/\hbar)^{\mathbf{2}} \, \langle \psi | \mathbf{a} \cdot \mathbf{S_A} \otimes \mathbf{b} \cdot \mathbf{S_B} | \psi \rangle.$$

Define $\cos \theta_{\mathbf{a},\mathbf{b}} \equiv \mathbf{a} \cdot \mathbf{b}$, then,

$$C(\mathbf{a}, \mathbf{b}) = -\cos \theta_{\mathbf{a},\mathbf{b}}$$

Choosing $\mathbf{a}, \mathbf{b}, \mathbf{a}', \mathbf{b}'$ to be four co-planar vectors with $\mathbf{a}$ and $\mathbf{b}$ parallel and $\phi \equiv \theta_{\mathbf{a},\mathbf{b}'} = \theta_{\mathbf{a}',\mathbf{b}}$, then the Bell inequality demands,

$$|1 + 2\cos(\phi) - \cos(2\phi)| \leq 2$$

but this is violated for a wide range of ϕ.

Observations:

The kind of locality that is violated by quantum mechanics is called *weak locality* because the violation does not permit 'super-luminal signaling.'

- That is, only a random sequence of outcomes are obtained at either location and the non-local correlations (on their own) can not be used to communicate information to the distant party.
- In contrast, a theory violating *strong locality* would allow the possibility of super-luminal signaling, e.g., rigid body mechanics.

Bell's argument relies also on an assumption of determinism: the outcomes are determined by the hidden variable $A = A(a, \lambda)$ and $B = B(b, \lambda)$.

- However, Clauser, Horne, Shimony, and Holt (1969), and Clauser and Horne (1974) developed inequalities (that assume local hidden variables and which are violated by quantum mechanics) that do not rely on the determinism assumption - only a probabilistic dependence on the hidden variables is presumed.

Some critics claim that Bell-type inequalities presume that the detector settings at the two separated locations may be selected independently, for example, by the 'free will' of the experimenters, or by some sufficiently pseudo-random function.

- But ultimately, in a fully deterministic conception of the world, all evens could be traced back to a common cause, and are never truly independent. This is super-determinism loop-hole suggests an even more bizarre conspiracy in which the outcomes of present day measurements were pre-determined at the big-bang so as to appear to violate the assumption of locality.

Are hidden variables non-local or is quantum mechanics non-local?
Bell-type inequalities tell us that any hidden variable models reproducing quantum theory must be non-local.

- However, if we reject hidden variable models, and assume instead that the quantum state is a complete description of a system's physical properties, then the EPR analysis shows (implicitly) that because quantum states must be updated (collapsed) non-locally, it follows that physical properties of the world are exhibiting non-locality.
- *So whether one accepts or rejects that quantum states are a complete description, one is forced to accept non-locality.*

- Some authors (Stapp, 1985, 1988) even conclude that sequences of experimental outcomes which violate Bell-type inequalities *imply* that non-locality is a feature of the world, rather than just a feature of quantum mechanics.
- In this sense the violation of Bell's inequalities should not be viewed as a reason to reject hidden variables, but as a required constraint on hidden variable models or any other reasonable ontological model.

The quantum mechanical violation of Bell-type inequalities has been demonstrated experimentally, in a number of distinct experiments.

- Some of the most important early experiments were performed by Alain Aspect and co-workers (1981,1982).
- Aspect gave lectures on his experimental tests of Bell-type inequalities for our course in 2005 and this year Gregor Weihs will present lectures to us on his experimental tests in February.