Interpretation of Quantum Theory, Phys 490/773
Problem Sheet 2
Due: March 14th

1. *Bell's Inequalities*.

   a) Derive Bell's inequalities for two particles and two independent detector settings for each particle. You should follow his 1971 argument or see, e.g., Ballentine's textbook. Interpret the main physical assumptions that are invoked to obtain this result.

   b) Consider some standard choice of detector directions (e.g., coplanar detector directions parameterized by a single angle θ). Simplify Bell's inequality for these settings.

   c) Calculate the quantum prediction for the correlations that are expected from the singlet state for the same settings as in problem 1.b).

   d) Graphically plot the quantum and local hidden variable results to illustrate the range and magnitude of the violation.

2. *de Broglie-Bohm Interpretation*.

   a) Using the polar decomposition $\psi = R\exp(iS/\hbar)$, Schrodinger's equation for $H = \vec{p}^2/2m + V(\vec{q})$, derive the partial differential equation satisfied by S and compare it to the Hamilton-Jacobi equation for classical dynamics.

   b) Assuming that the quantum particle's velocity is given by,

$$\frac{d\vec{x}}{dt} = \frac{-\nabla S(q,t)}{m}|_{\vec{q}=\vec{x}(t)}, \tag{1}$$

   derive the second order differential equation for the dynamics of a particle's position and write out explicitly how the new "quantum" potential Q depends on ψ.

   c) What is the continuity equation for the probability current?

3. *Wigner phase space distributions*.

   The Wigner function is a quantum analog of the probability distribution in "phase space". It is defined as the Fourier transform,

$$W(q,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} dy e^{2ipy}\rho(q-y, q+y) \tag{2}$$

   where $\rho(q-y, q+y) = \langle q-y|\rho|q+y\rangle$ is the density matrix in the coordinate representation and $\hbar = 1$.

   a) Show that if $\int \rho(q,q)dq = 1$, then $\int W(q,p)dqdp = 1$

   b) Show that $\int W_1(p,q)W_2(p,q)dqdp = \frac{1}{2\pi}Tr[\rho_1\rho_2]$ and therefore $\int W(p,q)^2 dqdp \leq \frac{1}{2\pi}$

   c) Plot $W(q,p)$ for $\rho(q,q') = \psi(q)^\dagger\psi(q')$ where $\psi(q) = (\frac{1}{\pi})^{1/4}e^{-q^2/2}$.

   d) Plot $W(q,p)$ for $\psi(q) = (\frac{\alpha}{\pi})^{1/4}e^{-\alpha q^2/2}$ for $\alpha = 4$ and $\alpha = 1/4$. Calculate the uncertainty in x and p, i.e., $\Delta x =< x^2 > - < x >^2$ and similarly for p, for both values of α. Interpret which features of the plot correspond to the uncertainty in x and p.

   e) Plot $W(q,p)$ for $\psi(q) = (\frac{1}{\pi})^{1/4}e^{-(q-q_0)^2/2}$ for $q_0 = 4$ and $q_0 = 10$. What is the physical difference between these two states?

f) Plot a$W(q,p)$ for the superposition of states $\psi(q) = (\frac{1}{4\pi})^{1/4}(e^{-(q-q_0)^2/2} + e^{-(q+q_0)^2/2})$, for $q_0 = 4$ and $q_0 = 10$. What property of the Wigner function is implied by the coherence between the two states in the superposition ? What happen as we increase q_0?

g) If the position q_0 is not known exactly but has a probability distribution of the form $p(\delta) = (\frac{\gamma}{\pi})^{1/2} e^{-\gamma\delta^2}$, the averaged Wigner function is given by

$$\bar{W}(\bar{q}, p) = \int d\delta p(\delta) W(\bar{q} + \delta, p).$$

Plot $W(q,p)$ for interesting values of γ for $\bar{q} = 10$.

4. *Term Project*.

Submit a brief (1 paragraph) description of your term project.