Interpretation of Quantum Theory	**U. Waterloo PHY773, Winter 2005**
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deBroglie-Bohm Interpretation	**Lectures 9 and 10**

1 The deBroglie-Bohm Interpretation

Lecture 9 Feb 1, 2005

The deBroglie-Bohm interpretation, which will be referred to here as *Bohmian Mechanics* (BM) was introduced in 1927 by deBroglie and rediscovered by Bohm in 1951/1952. It is in essence a reformulation of QM, and it can be seen as a formalism more so than an interpretation.

Why is BM needed? What is wrong with textbook orthodox QM? Not much is wrong, in some sense, but at the same time a lot of it is. In one regards it as a phenomenological formalism – an operational theory of experimental results – then it is very successful. As a fundamental theory, however – as it was originally intended to be a deeper physical theory at the microscopic level – it has many problems. The reason is that it is not clear what QM is about, what kind of object it describes, and what it says about those objects.

Is it about wave functions? What about the measurement problem and Schrödinger's cat? Superpositions are hard to interpret at the macroscopic level. Originally the Copenhagen interpretation was that QM was about measurements and observations, which is OK for a phenomenological theory, but as a fundamental theory it is problematic to be so anthropocentric, and measurement and observation are very vague notions – one would desire them to be explained *by* the theory, not fundamental concepts.

No-go theorems (such as Bell's) for local hidden-variables (LHV) theories seem to imply that measurement *creates* reality, while we would expect it to *reveal* reality.

A nice illustration if L. Hardy's version of the no-go theorem for LHV, where with 4 observables A,B,C and D, we have that

- the joint observation of outcomes $A = 1$ and $B = 1$ occurs some of the time
- if $A = 1$ occurs then $C = 1$ occurs
- if $B = 1$ occurs then $D = 1$ occurs
- but $C = 1$ and $D = 1$ never occur!

If measurements reveal reality, then these relationships should hold, which would lead to a conflict between what we would expect from simple logic and what we observe in the lab. If it created reality, each measurement creates a different reality, and there is no real problem.

So what is BM about? According to BM, QM is about particles – things with a definite location/position, just like in classical mechanics (in this sense, at least). The way with which they move is very different, however.

The state of a single particle is given by the configuration

$$\vec{Q} = (Q_1, Q_1, \cdots, Q_N) \tag{1}$$

which describes *the actual position of the particle*, and a function of the configuration space

$$\psi(q_1, q_2, \cdots, q_N) = \psi(\vec{q}) \tag{2}$$

which we call the wave function.

The equations of motion are

$$i\hbar\frac{d}{dt}\psi = \hat{H}\psi = \frac{\hbar^2}{2}\sum_k \frac{\nabla_k^2\psi}{m_k} + \hat{V}(q)\psi \tag{3}$$

$$\frac{d}{dt}\vec{Q}_k = \frac{\hbar}{m_k}\Im\left[\frac{\psi^*\nabla_k\psi}{\psi^*\psi}\right]_{q=Q}. \tag{4}$$

Unlike the regular approach to QM, BM does not need any axioms for measurements.

Note that the RHS of (4) can be seen simply as $\frac{\vec{J}_k}{\rho}$, where $\vec{J}_k$ is the probability current and $\rho = \psi^*\psi = |\psi|^2$. AS it is written in (4), it applies directly to particles with spin, where ψ is then a spinor valued wavefunction. if we cancel out the ψ^* the equation still work fine for spinless particles, but ψ^* is otherwise essential to keep the division well defined.

For spinless particles,

$$\frac{d}{dt}\vec{Q}_k = \frac{\hbar}{m_k}\Im\left.\frac{\nabla_k\psi}{\psi}\right|_{\vec{q}=\vec{Q}} \tag{5}$$

$$= \left.\frac{\nabla_k S}{m_k}\right|_{\vec{q}=\vec{Q}} \tag{6}$$

where $\psi = R\exp\left[i\frac{S}{\hbar}\right]$, which is the form originally written by deBroglie.

We also have a quantum continuity equation

$$\frac{d}{dt}\rho + \sum_k \nabla_k\cdot\vec{J}_k = 0. \tag{7}$$

Is there a separate wave function for each particle? **NO!** There is a single wave over the entire configuration space, which is not what one would expect intuitively, but it is the most general approach mathematically.

These equation of motion are deterministic. One needs:

- initial data for N particles, i.e. $\psi(q, t=0), \vec{Q}_1(t=0), \cdots, \vec{Q}_N(t=0)$.
- equations of motion.

NO OTHER AXIOMS ARE NECESSARY.

How does one reach (4), the guiding equation (GE)? There are many different approaches, and pretty much anything reasonable works. One approach is to simply day that

$$\vec{J} = \rho\vec{v} \rightarrow \vec{v} = \frac{\vec{J}}{\rho}. \tag{8}$$

Another approach is to require that

- the equations of motion be simple (first order).
- that they depend on ψ.
- that they have Galilean covariance.

and one obtains GE (especially in the spin 0 form).

What about the more celebrated QM equations?

$$E = h\nu \tag{9}$$

$$\vec{p} = \hbar\vec{k} \tag{10}$$

These two equations lead to Schrödinger's equation in a standard textbook way (not challenging). More importantly, they also lead to the GE, if we simply make the association $\vec{p} = m\vec{v}$, and by definition of a plane wave $\psi = \exp\left[i\vec{k}\cdot\vec{q}\right]$ one can obtain $\vec{k}$ simply by $\vec{k} = \frac{\nabla_k \psi}{\psi}$.

What about Feynman's criticism in the double-slit experiment, that the particle knows that the other slit is open or closed? Well, the wave function is a function of the whole configuration so it knows that the slit is open – it knows the whole configuration at the same time.

So what is the relationship between BM and orthodox QM? There is an empirical equivalence (same predictions), but BM also *implies* QM (but the converse is not true). In particular, as consequences to BM we have

- spectral lines
- operators as observables
- scattering theory
- collapse of wave function open measurement
- absolute uncertainty/Heisenberg-Robertson uncertainty principle/quantum uncertainty.

By *absolute uncertainty* it is meant that in a Bohmian universe that is in quantum equilibrium, i.e. $\rho = |\psi|^2$, it is impossible to know more about the system than it's wave function allows – that is, the precision of the measurement is constrained by the spread of ρ. **Lecture 10** Feb 3, 2005

2 Quantum Randomness

The quantum probability density $\rho = |\psi|^2$ is special and has many unique properties. In the correspondence $\psi \to \rho^\psi$, if we evolve ψ into $\psi_t \to \rho^{\psi_t}$, we obtain the same probability distribution as if we had let ρ^ψ evolve into $\left(\rho^\psi\right)_t$, that is

$$\rho^{\psi_t} = \left(\rho^\psi\right)_t. \tag{11}$$

This is what we call *equivariance*. If Q_t is $|\psi_t(q)|^2$ distributed at one time t, then Q_t is $|\psi_t(q)|^2$ at all times. Equivariance is a natural extension of stationarity in autonomous systems.

But why does equivariance hold? Probabilities flow the same way that fluids flow, that is

$$\frac{d}{dt}\vec{Q} = \vec{v} \tag{12}$$

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0, \tag{13}$$

even if $\vec{v}$ and ρ are time dependent.

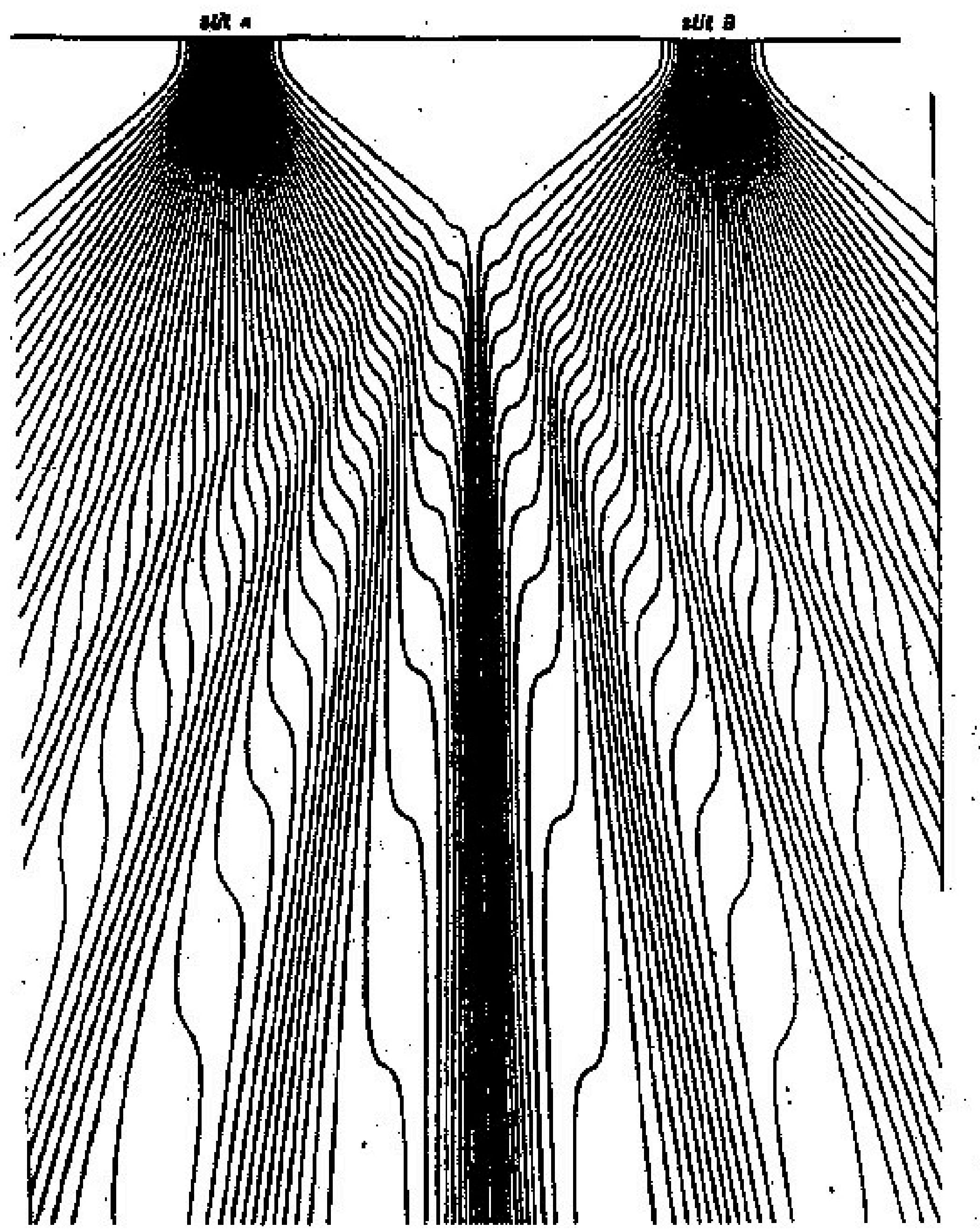

Figure 1: The double-slit experiment.

3 Hidden Variable Theories and Bohmian Mechanics

We claim that BM is a counter example to any no-go theorem for HVs. Consider a system and an apparatus such that

$$t = 0 \qquad \Psi, Q \qquad \Pr(q = Q) = |\Psi(Q)|^2$$

$$t = T \qquad \Psi_T, Q_T \qquad \Pr(q = Q) = |\Psi_T(Q)|^2.$$

This BM model makes exactly the same predictions that QM makes, so in that sense it is a counter example to no-go theorems for HVs. So we may say that BM is equivalent to orthodox QT modulo $|\psi|^2 = \rho$, that is, if the initial distribution is $|\psi|^2 = \rho$ then there is complete equivalence. This is called the *quantum equilibrium distribution*.

When does this condition hold? This is the problem of *quantum equilibrium*.

Since one cannot realistically decouple the subsystems, BM can only be thought to describe the whole universe. In this sense, how can we talk about the probability density of the universe? We only have one sample! So what could this probability density mean?

If it describes only a subsystem, we have to say what it means to have only a partial description of the wave function.

One way to think about quantum equilibrium (QE), is to take it to be analogous to the probability density of particles in a gas

$$\text{QE} \sim f_T \sim \exp\left[-\frac{mv^2}{kT}\right]. \tag{14}$$

Both ideas are controversial, but with the same controversy. Another way to think about it is as with dynamical systems, where the random behavior is governed by a stationary probability.

Consider a Bohmian universe with configuration Q of all the particles in this universe. We can decompose it $Q = (X, Y)$, where X is the system of interest, and everything else, Y, we call the environment. The whole space is decomposed similarly $q = (x, y)$, so the state is given by $\Psi(x, y), X, Y$. We want $\Psi(x)$, that is, a wave function of the system of interest only. This is, in fact, given by

$$\Psi(x) = \Psi(x, Y), \tag{15}$$

where Y is the actual configuration of the environment. This is the conditional wave function. It has the same equations of motion, i.e. Schrödinger's equation and the GE

$$\frac{d}{dt}X = v^{\Psi}(x). \tag{16}$$

What about the conditional probability? It turns out to be

$$\Pr(X_t \in dx | Y_t) = |\Psi_t(x)|^2 dx, \tag{17}$$

as we would expect from the conditional wave function.

This says that no matter how much we know about the environment, we are *still* limited by the distribution given by $|\Psi(x)|^2$.

The time dependence of the conditional wave function comes from the time dependence of the universe's wave function *and* from the time dependence of the environment configuration Y, and this is how we obtain the collapse of the wave function.

To understand the collapse, consider a system and apparatus with the initial wave function

$$\Psi_0 = \psi(x)\Phi_0(y). \tag{18}$$

Let $\psi(x) = \sum c_\alpha \psi_\alpha$, that is, we decompose it into eigenfunctions of an observable. In the measurement process, we let the system interact with the environment, so that

$$\Psi_0 \to \sum c_\alpha \psi_\alpha(x)\Phi_\alpha(y) = \Psi_T(x, y). \tag{19}$$

What happens to the conditional wave function? Initially it is $\psi(x)$ at $t = 0$. What is $\psi_T(x) = \Psi_T(x, Y_T)$? It is determined by Y_T, which can only be one of the alpha, so that $\Phi_{\alpha'} = 0$ for $\alpha' \neq \alpha$, and thus, $\psi_T = \psi_\alpha$ upon measurement, with probability $|c_\alpha|^2$. That is, we have the collapse of the wave function in the same way as orthodox QT.

4 Non-locality and hidden variables

Usually it is stated that the price to pay for a HV theory is non-locality. This is not quite correct. Bell's theorem was made of two parts

i EPR contribution (or the simpler version by Bohm)

Take the predictions of QM with regard to, say, spin correlations, and assume locality. The result is that ψ is not a complete description of reality (i.e. there are HVs).

ii Bell contribution

Take again, spin correlations, as predicted by QM, and one obtains a no-go theorem from HVs. Note that there is no locality assumption.

What is the conclusion? There is a contradiction, so one of the assumptions must be wrong. We say that QM is right due to the experimental verifications of its predictions. Thus, *locality* is the wrong assumption. QM is non-local, and Nature is non-local.

5 Properties of quantum observables

What properties does a Bohmian particle of system have? Do they *only* have position? What about momentum, spin, etc. ?

Consider the ground stated of an H atom. It is initially real and has some spread, and its Fourier transform is similarly spread. Thus, momentum is spread out. However, $\frac{d}{dt}Q = \Im(\cdots) = 0$. The velocity distribution is a Dirac delta at the origin! Is there an empirical contradiction? No, because we need to consider the outcome of measurements. Taking measurements into account, the apparent contradiction is resolved.

What about general observables? Start out with the system in state ψ and the apparatus in state Ψ_0, that is, $\psi \otimes \Psi_0$, and allow for some interaction U_T between the two, resulting in Ψ_T for the system and the apparatus. We are interested in the distribution of the result $Z = F(Q_T)$. Now, $\rho_{\psi_T} = |\Psi_T|^2$. The composition of maps $\rho_{\psi_T} \circ F^-1$ is what we want. This almost all the maps we considered are linear, aside from $|\cdot|^2$. This, the mapping from ψ to a distribution of general measurement outcomes is quadratic, so the probability can be mapped by

$$\psi^\dagger E(dZ)\psi. \tag{20}$$

Thus for all positive operator valued measures are given by quadratic maps. So this is how operators as observables come into BM – nothing but a mathematical convenience.

In general, no-go theorems for HVs say: there is no good map from a set of observables A to a random variable $X_A(\omega)$ over a countable space. By "good" we mean that the joint distribution should match QM – in other words, "good" is empirical agreement with experiments.

How does BM escape this? Consider an experiment $\mathcal{E}$, and a distribution Z^{ψ} of outcomes, all associated with the same observable A. So we have a map from Z^{ψ}, a random variable, to A, a quantum observable. This is a map in the opposite direction from the one in no-go theorems for HVs, so those theorems do not apply to BM.

So the problem is assuming that the operator is enough to specify the measurement. This is the naïve realism about operators we are taught in orthodox QM. This is not a reasonable assumption, and we should avoid it.

6 $(\psi, X), (\psi, \textbf{?})$

Einstein said ψ was not a complete description of reality. BM says that in order to obtain a complete description of reality, we just need the particle positions. The question mark is about whether it is possible that the "remaining" description of reality that completes QM could be something else other than the particle position.

As an extreme example, one may consider that there are no world-lines, but "flashes". This actually works, and it is fully Lorentz invariant – R. Tumulka did this as an extension of Bell's work.

Thus, the simple/natural choice may not be the best. For BM, it is crucial to have X, the configuration. This is what the theory is all about. The right X might deserve a role in the development of future possibilities for physics – we need to think about what ontologies are.